8.5 Solving Exponential Equations: Applications

Recall:

$f(x) = ab^x$ → the number of growth/decay periods

final amount (or number)

initial amount (or number)

b is the multiplier $[b = 1 \pm r_{ate}]$

$b = 1 + r \Rightarrow$ exponential growth (r as a decimal)

$b = 1 - r \Rightarrow$ exponential decay

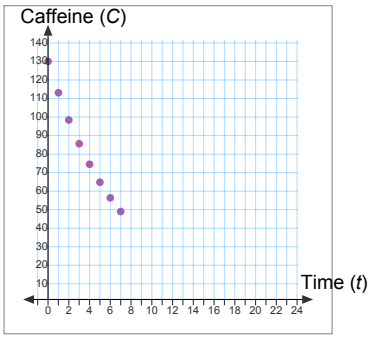
example

2% growth: $b = 100\% + 2\% = 102\% = 1.02$

2% decay: $b = 100\% - 2\% = 98\% = 0.98$

Caffeine is present in most coffees, teas, chocolates and several other food and beverage products. Research shows that this chemical is eliminated from the human body over time exponentially (mostly). We will verify this using observed data.

The table and graph show the mass of caffeine (C) remaining in an average-sized person, for 7 hours after drinking a cup of coffee that contained 130 mg of caffeine.



t (hours)	C (mg)	y-ratios
0	130	
1	113.1	$\frac{113.1}{130} = 0.87$
2	98.4	$\frac{98.4}{113.1} \doteq 0.8700$
3	85.6	$\frac{85.6}{98.4} \doteq 0.8699$
4	74.5	
5	64.8	
6	56.4	
7	49	$\frac{49}{56.4} \doteq 0.8687$

If C represents mass of caffeine remaining, in mg, and t represents the time, in hours, after drinking the coffee, calculate the half-life of caffeine, to the nearest hundredth.

$\therefore b \doteq 0.87$ (multiplier)

(comparisons on next page)

$f(x) = ab^x$

$C(t) = ab^t$

$C(t) \doteq 130(0.87)^t$

∴ when $C(t) = 65$, t is the half-life

$65 \doteq 130(0.87)^t$

$\frac{65}{130} \doteq 0.87^t$

$\frac{1}{2} \doteq 0.87^t$

$\log \frac{1}{2} \doteq \log .87^t$

$\log \frac{1}{2} \doteq t \log 0.87$

$\frac{\log \frac{1}{2}}{\log 0.87} \doteq t$

$t \doteq 4.977$

$\doteq 4.98$ hours

∴ the half-life of caffeine is 4.98 hours.

Alternate Method

$M(t) = P(\frac{1}{2})^{\frac{t}{h}}$

$C(t) = 130(\frac{1}{2})^{\frac{t}{h}}$

$C(2) = 98.4$

$98.4 = 130(\frac{1}{2})^{\frac{2}{h}}$

$\frac{98.4}{130} = (\frac{1}{2})^{\frac{2}{h}}$

$\log(\frac{98.4}{130}) = \frac{2}{h}\log(\frac{1}{2})$

$h\log(\frac{98.4}{130}) = 2\log(\frac{1}{2})$

$h = \frac{2\log(\frac{1}{2})}{\log(\frac{98.4}{130})}$

$\doteq 4.977$

$\doteq 4.98$ hours

http://www.timhortons.com/ca/en/menu/nutrition-and-wellness-resources.php

Internet "Facts"

Tim Horton's Coffee size vs. Caffeine content

small	(10 oz.)	140 mg
medium	(14 oz.)	205 mg
large	(20 oz.)	270 mg
X-large	(24 oz.)	330 mg

Starbucks

410 mg venti (20 oz.)

For comparison, a 8.4 oz can of Red Bull has 80 mg.

For comparison, a 12 oz can of Coca-Cola has 34 mg.