

Before we begin, are there any questions from last day's work?

pp.370-372 #1, 2, 5, 6, 12b(i-iii)

Worksheet 1.1.3

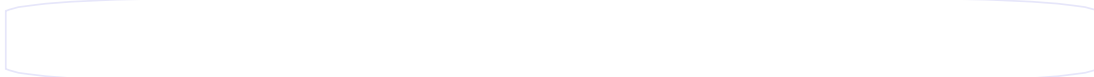
Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) solve simple exponential equations numerically and graphically, with technology
- b) connect algebraic and graphical representations of exponential equations.

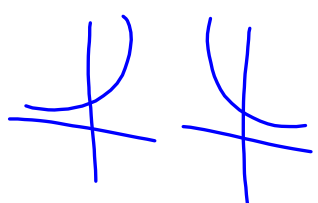
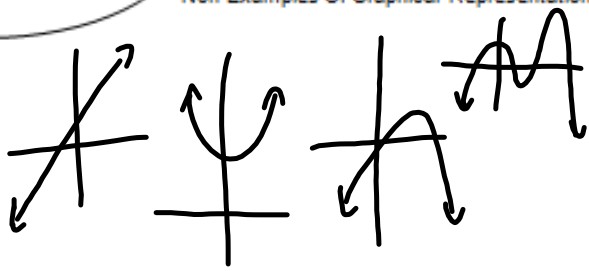
Day 1: We will work through "Mystery 1" and "Mystery 2".

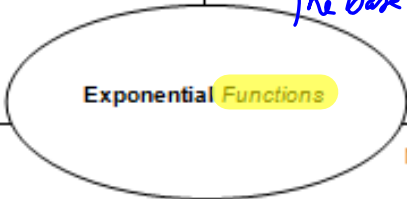
Day 2: We will work through "Mystery 3" and "Mystery 4".



1.2.1: Frayer Model – Exponential Functions

Date: Feb 8/18

| | |
|---|--|
| <p>Examples Of Algebraic Representations</p> <p>$y = 2^x$; $y = \left(\frac{1}{2}\right)^x$; $y = 3(2.7)^x$; $y = 2^{-x}$</p> <p>Non-Examples Of Algebraic Representations</p> <p>$y = 3x - 7$</p> | <p>Characteristics</p> <p>The variable <u>must</u> be in the exponent.</p> <p>The base must be $0 < b < 1$ or $b > 1$</p> <p>NB: $b \neq 0$ [or line $y = 1$]</p> |
| <p>Examples Of Graphical Representations</p>  | <p>Non-Examples Of Graphical Representations</p>  |



1.2.2: Can You Solve This Mystery?

Date: _____

Recall: "To solve an equation" means to determine the value of the variable that makes the equation true.

MYSTERY #1 – How can you solve exponential equations?

1. Solve these exponential equations. Match the solution with the equation.

| Equation | | Solution | |
|---------------|------------------------|----------|----------|
| <u> </u> C | 1. $2^x = 8$ | A. | $x = -2$ |
| <u> </u> D | 2. $2^x = 16$ | B. | $x = 0$ |
| <u> </u> A | 3. $2^x = \frac{1}{4}$ | C. | $x = 3$ |
| <u> </u> B | 4. $2^x = 1$ | D. | $x = 4$ |

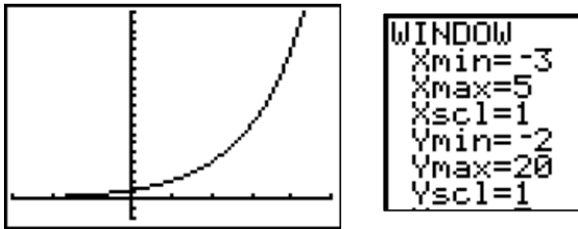
2. Describe the process that you used to solve the equations above.

- 👉 trial and error
- 👉 inspection
- 👉 substitution

3. Solve
- $2^x = 5$
- . Round your answer to the nearest hundredth.

👉 $x \doteq 2.32$

4. Use a graphing calculator and window settings to graph the function
- $y = 2^x$
- .



5. Complete the following statements.

- a) The value of the function is 16 when $x =$ 4 👉
- b) The value of the function is $\frac{1}{4}$ when $x =$ -2 👉
- c) The value of the function is 8 when $x =$ 3 👉
- d) The value of the function is 5 when $x =$ 2.32 👉 (round your answer to two decimal places)

6. Explain how to use the graph of the function
- $y = 2^x$
- to solve the equations in #1.

👉 trace, number [(x =) appears], enter

7. Explain why you
- can not**
- use the graph of the function
- $y = 2^x$
- to solve the equation
- $2^{2x-3} = 8$
- but you
- can**
- use
- $y = 2^x$
- to solve the equation
- $2^x = 8$
- .

👉 As you "trace" along $y=2^x$, you are getting values of x , not $2x-3$.👉 Also, $y=2^x$ does not represent $y=2^{2x-3}$.

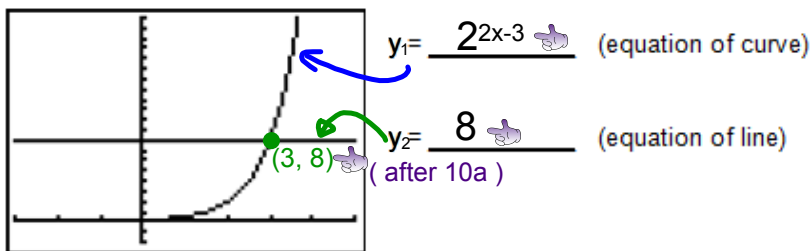
MYSTERY #2 – How can you solve more difficult exponential equations?

8. Fear not! There is a way to solve the equation $2^{2x-3} = 8$.

Enter the left side of the equation as one function and the right side of the equation as another function using the $\boxed{Y=}$ editor of your calculator as shown.

| Plot1 | Plot2 | Plot3 |
|--------------------|-------|-------|
| $Y_1 = 2^{(2X-3)}$ | | |
| $Y_2 = 8$ | | |
| $Y_3 =$ | | |
| $Y_4 =$ | | |

9. Label the functions as $y = 2^{2x-3}$ and $y = 8$ on the screen shot below.



10. Use the intersect operation to find the point of intersection of the two functions and complete the following statements. (for all)

a) The point of intersection occurs when $x = 3$ and $y = 8$.

The solution to the equation is $x = 3$.

When $x = 3$, both functions have a value of 8 .

Day 2: We will work through "Mystery 3" and "Mystery 4".

MYSTERY #3

11. Use your graphing calculator and the "Intersection Method" to solve the following equations. Record the solution.

| Equation | Y_1 | Y_2 | Solution to Equation |
|------------------------|----------------------|------------|----------------------|
| a) $2^{2x-3} = 8$ | $Y_1 = 2^{2x-3}$ | $Y_2 = 8$ | $x = 3$ |
| b) $2^{2x-3} - 6 = 2$ | $y_1 = 2^{2x-3} - 6$ | $y_2 = 2$ | $x = 3$ |
| c) $2^{2x-3} + 5 = 13$ | $y_1 = 2^{2x-3} + 5$ | $y_2 = 13$ | $x = 3$ |

12. Solve the mystery... Why are all of the solutions to these equations the same?

$$\begin{aligned}
 \text{b) } 2^{2x-3} - 6 &= 2 \\
 2^{2x-3} &= 2 + 6 \\
 2^{2x-3} &= 8
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } 2^{2x-3} + 5 &= 13 \\
 2^{2x-3} &= 13 - 5 \\
 2^{2x-3} &= 8
 \end{aligned}$$

MYSTERY #4

13. True or false (check one)? "The solution to any exponential equation is always an exact value."
 true or false Justify your choice.

$$2^x = 5$$

$$x \approx 2.321$$

14. Use trial and error on your calculator to determine the solution to the following equations. Compare your solution using the "Intersection Method".

| Equation | Solution Using Trial And Error On <i>Your</i> Calculator (2 decimal places) | Solution Using The Intersection Method (<u>3</u> decimal places) |
|-------------------|---|---|
| a) $3.05^x = 15$ | $x =$ | $x \approx 2.428$ |
| b) $2.3^x = 6$ | $x =$ | $x =$ |
| c) $4.2^{-x} = 7$ | $x =$ | $x =$ |
| d) $2.7^x = 38$ | $x =$ | $x =$ |

15. Make an exponential equation, with base 5, where the solution is

| | |
|--|--|
| <p>a) an exact value</p> $\left. \begin{array}{l} 5^x = 25 \\ x = 2 \end{array} \right\} \begin{array}{l} 5^x = 125 \\ x = 3 \end{array}$ <hr/> $5^x = 0.04 \text{ or } 5^x = \frac{1}{25}$ $x = -2$ <p style="text-align: center; color: red;">same</p> | <p>b) 0.75</p> $5^x \approx 3.34$ $x = 0.75$ |
| <p>c) a negative integer</p> $5^x = 0.008$ $x = -3$ | <p>d) an irrational number (not exact)</p> $5^x = 32$ $x \approx 2.15338279$ |

$$\mathbb{R} = \{\mathbb{Q} \cup \bar{\mathbb{Q}}\}$$

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0 \right\}$$

$$\mathbb{Z} = \{ \dots, -2, -1, 0, 1, 2, \dots \}$$

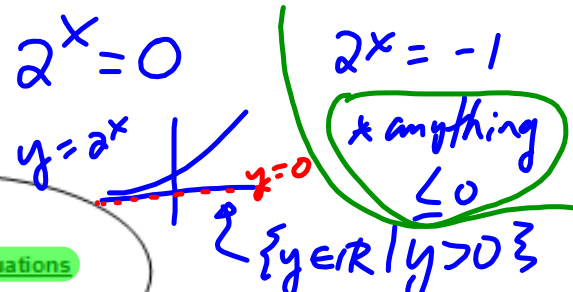
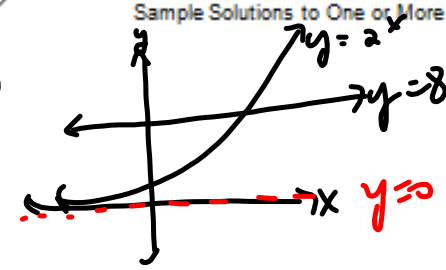
$$\mathbb{W} = \{ 0, 1, 2, 3, \dots \}$$

$$\mathbb{N} = \{ 1, 2, 3, \dots \}$$

$$\bar{\mathbb{Q}}$$
$$\pi, e$$
$$\sqrt{2}, \sqrt{3}$$
$$5.35335333\dots$$

1.2.3: Frayer Model – Exponential Equations

Date: _____

| | |
|---|---|
| <p>Examples Of Solvable Equations</p> $2^x = 8 \quad 3^x = 27$ $2^x = 9.2 \quad 3^x = 28$ | <p>Examples of Non-Solvable Equations</p> $2^x = 0$  $2^x = -1$ <p>* anything ≤ 0</p> <p>$\{y \in \mathbb{R} \mid y > 0\}$</p> |
| <p>Exponential Equations</p> | |
| <p>Strategies</p> <p>inspection</p> <p>trial & error</p> <p>T184 intersection method</p> <p>↳ graphically</p> | <p>Sample Solutions to One or More</p>  $2^x = 8$ $y_1 = 2^x$ $y_2 = 8$ |

$$2^x = 8$$

$$2^x = 2^3$$

$$\therefore x = 3$$

1.2.4: Further Mysteries to Ponder

Date: _____

- Using only a regular calculator, use trial and error to determine the solution to the equation $1.05^x = 1276$ (to 3 decimal places).
- Explain why $5^x = -2$ has no solution, but $2^x - 5 = -2$ has a solution. Provide graphical and algebraic support.
- Is it possible to create an exponential equation that has more than one solution? Provide graphical and algebraic support.
- Compare and contrast the solutions in group A with those in group B.

| Group A | Group B |
|------------------------|---------------------|
| $2^{-x} = 8$ | $2^x = 8$ |
| $2^{-x} = 1$ | $2^x = 1$ |
| $2^{-x} = \frac{1}{4}$ | $2^x = \frac{1}{4}$ |
| $2^{-x} = 16$ | $2^x = 16$ |