Before we begin, are there any questions from last day's work?

pp. 329-331 # 7, 11, 2, 4, 5

Extra Practice p.330 # 9, 10

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use the exponent laws to simplify and evaluate expression

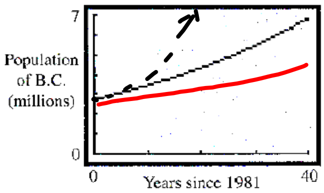
b) solve exponential equations by using common bases.

1.4.2: Applications of Exponential Functions pp. 329-331 # 7, 11, 2, 4, 5

2. The population, P million of British Columbia can be modelled by the equation $P = 2.76(1.022)^n$, where n is the number of years since 1981.

$P = P_0(1 \pm r)^n$

a) Use the equation to estimate the annual rate of growth, as a percent.
b) Describe how both the graph and the equation change in each case.
   i) The growth rate increases. ii) The growth rate decreases.



$b = 1 \pm r$

$\therefore b = 1.022$

$\therefore r = 0.022$

$\therefore r = 2.2\%$

bi) if $r \uparrow$

$P = 2.76(1.023)^n$

graph increase more quickly

ii) if $r \downarrow$, but > 0

ie) $r = 1.0000\_$

$P = 2.76(1.002)^n$

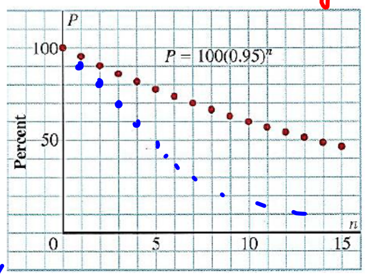
graph increases more slowly

4. Several layers of glass are stacked together. Each layer reduces the light passing through it by 5%. The percent, P, of light passing through n layers is represented by the equation $P = 100(0.95)^n$.

Again, n, is a natural number because it indicates the number of layers of glass.

a) Estimate how many panes are needed before only 50% of the light passes through.

b) The graph was drawn for clear glass. For frosted glass, each layer reduces the light passing through it by 10%. Describe how both the graph and the equation changes for this glass.



$P = 100(1-0.1)^n$

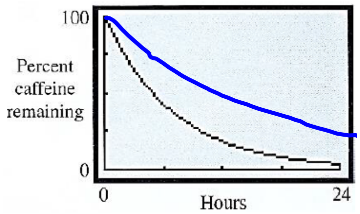
$= 100(0.9)^n$

100% − 10% = 90% = 0.9

100% − 5% = 95% = 0.95

5. When you consume caffeine, the percent, P, left in your body can be modelled as a function of the elapsed time, n hours, by the equation $P = 100(0.87)^n$.

Describe how both the graph and the equation may change for pregnant women, who required a much longer time to metabolize caffeine than other adults.



11. For every metre below the surface of water, the light intensity is reduced by 2.5%

a) Write an equation to express the percent, P, of light remaining as a function of the depth, d metres, below the surface.

b) Graph P as a function of d.

c) Describe how the graph would is similar to, and different from, the graph from #4 (above) that shows the percentage of light transmitted through panes of glass.

d) Determine the light intensity at a depth of 10 m.

e) At what depth is the light intensity reduced to 50% of the intensity at the surface?

a) $b = 1 \pm r$ $\quad r = 2.5\%$

$\therefore b = 1 - r$ $\quad = 0.025$

$= 1 - 0.025$

$= 0.975$

$\therefore P = 100(0.975)^d$

1.5.1: Simplifying and Evaluating Expressions Using the Laws of Exponents

Date: Feb. 13/18

Ex. 1 Evaluate without using a calculator. [You must use the laws of exponents]

a) 3^{-2} b) $\left(\frac{1}{4}\right)^{-2}$ c) $\frac{1}{4^{-2}}$ d) $\left(\frac{3}{4}\right)^{-2}$ e) $\left(-\frac{2}{5}\right)^{-3}$ f) $\left(\frac{81}{16}\right)^{\frac{1}{2}}$ g) $27^{\frac{2}{3}}$ h) $64^{\frac{4}{3}}$ i) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

a) $=\left(\frac{1}{3}\right)^2 = \frac{(1)^2}{(3)^2} = \frac{1}{9}$

b) $=\left(\frac{4}{1}\right)^2 = 4^2 = 16$

c) $=1\cdot 4^2 = 16$

d) $=\left(\frac{4}{3}\right)^2 = \frac{(4)^2}{(3)^2} = \frac{16}{9}$

e) $=\left(\frac{-5}{2}\right)^3 = \frac{(-5)^3}{(2)^3} = \frac{-125}{8}$

f) $=\frac{\sqrt[2]{81}}{\sqrt[2]{16}} = \frac{9}{4}$

g) $=\left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

h) $=\left(\sqrt[3]{64}\right)^4 = 4^4 = 256$

i) $=\frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$

Ex. 2 Simplify using the laws of exponents.

a) $\sqrt{x^6y^{12}}$

$=\left(x^6y^{12}\right)^{\frac{1}{2}}$

$=\left(x^6\right)^{\frac{1}{2}}\left(y^{12}\right)^{\frac{1}{2}}$

$=x^3y^6$

b) $x^2 \div x^{\frac{3}{2}}$

$=x^{2-\frac{3}{2}}$

$=x^{\frac{4}{2}-\frac{3}{2}}$

$=x^{\frac{1}{2}}$

$=\sqrt{x}$

c) $\left(x^{\frac{2}{5}}\right)^{\frac{5}{8}}$

$=x^{\frac{\cancel{2}^{1}}{\cancel{5}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{8}_{4}}}$

$=x^{\frac{1}{4}}$

$=\sqrt[4]{x}$

d) $\sqrt{81x^{16}y^{100}}\left(2x^{-4}y^3\right)^2$

$=\left(81x^{16}y^{100}\right)^{\frac{1}{2}}\left[(2)^2\left(x^{-4}\right)^2\left(y^3\right)^2\right]$

$=81^{\frac{1}{2}}\left(x^{16}\right)^{\frac{1}{2}}\left(y^{100}\right)^{\frac{1}{2}}\left[4x^{-8}y^6\right]$

$=9x^8y^{50}\cdot 4x^{-8}y^6$

$=36x^{8+(-8)}y^{50+6}$

$=36x^0y^{56}$

$=36y^{56}$

Ex. 3

a) Simplify $\dfrac{a^3b^2c^3}{\sqrt{a^2b^4}}$, and then evaluate for $a=4$, $b=9$, and $c=-3$.

$$= \frac{a^3b^2c^3}{(a^2b^4)^{\frac{1}{2}}}$$

$$= \frac{a^3b^2c^3}{ab^2}$$

$$= a^{3-1}b^{2-2}c^3$$

$$= a^2c^3$$

$$= (4)^2(-3)^3$$

$$= 16(-27)$$

$$= -432$$

b) Verify your answer by evaluating the expression ***without*** simplifying first.

$$= \frac{(4)^3(9)^2(-3)^3}{\sqrt{(4)^2(9)^4}}$$

$$= \frac{64(81)(-27)}{\sqrt{16(6561)}}$$

$$= \frac{-139\,968}{\sqrt{104\,976}}$$

$$= \frac{-139\,968}{324}$$

$$= -432$$

1.5.2: Solving Exponential Equations Using Common Bases

Date: Feb.13/18

Ex. 1 Solve each exponential equation by determining a common base.

a) $2^x = 32$

$2^x = (2)^5$

$\therefore x = 5$

b) $3^{5x+8} = 27^x$

$3^{5x+8} = (3^3)^x$

$3^{5x+8} = 3^{3x}$

$\therefore 5x+8 = 3x$

$5x - 3x = -8$

$2x = -8$

$x = -4$

c) $3^{2x+5} = 27^{4x}$

$3^{2x+5} = (3^3)^{4x}$

$\therefore 2x+5 = 12x$

$2x - 12x = -5$

$-10x = -5$

$x = \frac{-5}{-10}$

$x = \frac{1}{2}$

d) $4^{5x-1} = 2^{2(x+11)}$ **

$(2^2)^{5x-1} = (2^2)^{x+11}$

$\therefore 2(5x-1) = 2(x+11)$

$10x - 2 = 2x + 22$

$10x - 2x = 22 + 2$

$8x = 24$

$x = 3$

e) $4^{3x} = 8^{x+1}$

$(2^2)^{3x} = (2^3)^{x+1}$

$2^{6x} = 2^{3x+3}$

$\therefore 6x = 3x + 3$

$3x = 3$

$x = 1$

f) $3(2^{x-1}) = 96$

$\frac{3(2^{x-1})}{3} = \frac{96}{3}$

$2^{x-1} = 32$

$2^{x-1} = 2^5$

$\therefore x - 1 = 5$

$x = 6$

g) $5(3^{x+3}) = 405$

$3^{x+3} = \frac{405}{5}$

$3^{x+3} = 81$

$3^{x+3} = 3^4$

$x = 1$

h) $\sqrt{2} = 4^{x+1}$

$2^{\frac{1}{2}} = (2^2)^{x+1}$

$2^{\frac{1}{2}} = 2^{2x+2}$

$\therefore \frac{1}{2} = 2x + 2$

$\frac{1}{2} - 2 = 2x$

$\frac{1}{2} - \frac{4}{2} = 2x$

$-\frac{3}{2} = 2x$

$-\frac{3}{4} = x$

i) How could you check your solutions using graphing technology?

Answer: Use the intersection method; i.e. $y_1 = 2^x$, $y_2 = 32$

Homework: p.387 #1, 2a, 3a, 5, 6
Worksheet 1.5.3

1.5.3 Solving Exponential Equations Using Common Bases Date: ________________

1. Solve each exponential equation by determining a common base.

a) $2^x = 64$ b) $5^{2x+6} = 125$ c) $5^x = \frac{1}{25}$ d) $4^x = \frac{1}{8}$

2. Simplify.

a) $\frac{27^3 \times 9^{-2}}{81}$ b) $\frac{25^{m+3n}}{125^{2m+1}}$

$= \frac{(5^2)^{m+3n}}{(5^3)^{2m+1}}$

$= 5^{2m+6n} \div 5^{6m+3}$

$= 5^{2m+6n-(6m+3)}$

$= 5^{2m+6n-6m-3}$

$= 5^{-4m+6n-3}$

$= 5^{6n-4m-3}$

3. Solve.

a) $3(2^x) = 48$ b) $4(7^{2x-1}) = 28$ c) $9^{x+1} = 27^{3x-4}$ d) $2^{2x+4} - 5 = 59$

4. Solve.

a) $2^{x^2+5x} = 64$ b) $(3^{x-3})^x = \frac{1}{9}$ c) $3^{3x+1} = 27(9^x)$ d) $(2^{x+2})(4^{x-1})(8^{2x-3}) = 256^x$

a)
$2^{x^2+5x} = 2^6$
$\therefore x^2+5x = 6$
$x^2+5x-6=0$
$(x+6)(x-1)=0$
$\therefore x=-6, 1$

b)
$3^{x^2-3x} = (3^{-2})$
$\therefore x^2-3x=-2$
$x^2-3x+2=0$
$(x-2)(x-1)=0$
$\therefore x=2, x=1$

c)
$3^{3x+1} = (3)^3(3^2)^x$
$3^{3x+1} = 3^3 \cdot 3^{2x}$
$3^{3x+1} = 3^{3+2x}$
$\therefore 3x+1 = 3+2x$

d) $(2^{x+2})(4^{x-1})(8^{2x-3}) = 256^x$

$(2^{x+2})((2^2)^{x-1})((2^3)^{2x-3}) = (2^8)^x$

$(2^{x+2})(2^{2x-2})(2^{6x-9}) = 2^{8x}$

$2^{x+2+2x-2+6x-9} = 2^{8x}$

$\therefore 9x-9 = 8x$

$9x-8x=9$

$x=9$

Answers

1a) 6	b) $\frac{-3}{2}$	c) -2	d) $\frac{-3}{2}$	2a) 3	b) $5^{6n-4m-3}$
3a) 4	b) 1	c) 2	d) 1	4a) -6 or 1	b) 2 or 1	c) 2	d) 9