Before we begin, are there any questions from last day's work?

4c

7

6bc

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use a cubic model to solve a problem *with* technology.

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4. A company that charters a boat for tours around the Gulf Islands can sell 200 tickets at \$50 each. For every \$10 increase in the ticket price, 5 fewer tickets will be sold.

a) Represent the number of tickets sold as a function of the selling price.

b) Represent the revenue as a function of the selling price.

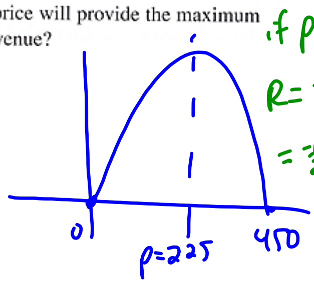
c) Sketch the function. What selling price will provide the maximum revenue? What is the maximum revenue?

b) $R = -\frac{1}{2}p^2 + 225p$

c) Let $R = 0$

$\therefore 0 = -\frac{1}{2}p(p-450)$

$\therefore p = 0$ or $p = 450$



if $p = 225$

$R = -\frac{1}{2}(225)(225-450)$

$= -\frac{1}{2}(225)(-225)$

$= 25312.50$

$\therefore$ a selling price of \$225 will provide a max revenue of \$25312.50

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6. Computer programs are sold to students for \$25 each. Two hundred students are willing to buy them at that price. For every \$5 increase in price, there are 20 fewer students willing to buy the software.

a) Represent the sales revenue as a function of the price. Sketch the function.

b) What is the maximum revenue?

c) What range of prices will give a sales revenue that exceeds \$5400?

a) $S = -\frac{20}{5}p^2 + 300p$

$= -4p^2 + 300p$

$S = -4p(p-75)$

b) $S = -4p^2 + 300p$

$= -4(p^2 - 75p)$

$= -4(p^2 - 75p + 37.5^2 - 37.5^2)$

$= -4(p-37.5)^2 - 4(1406.25)$

$= -4(p-37.5)^2 + 5625$

$\left(\frac{1}{2}b\right)^2$

$\left(\frac{1}{2}(-75)\right)^2$

$= (37.5)^2$

$= 1406.25$

$\therefore$ max revenue is \$5625

c) $5400 = -4p^2 + 300p$

$4p^2 - 300p + 5400 = 0$

$4(p^2 - 75p + 1350) = 0$

$4(p-30)(p-45) = 0$

$\therefore p = 30$ or $p = 45$

$\therefore$ if $30 < p < 45$, then sales revenue will exceed \$5400.

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7. The daily profit, P dollars, of a cotton candy vendor at the fair is described by the function $P = -60x^2 + 240x - 80$, where x dollars is the selling price of a bag of cotton candy.

a) What should the selling price of a bag of cotton candy be to maximize daily profits?

b) What is the maximum daily profit?

$P = -60x^2 + 240x - 80$

$= -60(x^2 - 4x) - 80$

$= -60(x^2 - 4x + 4 - 4) - 80$

$= -60(x-2)^2 - 60(-4) - 80$

$= -60(x-2)^2 + 240 - 80$

$= -60(x-2)^2 + 160$

a) a selling price of \$2 per bag maximizes profit.

b) the max. profit is \$160.

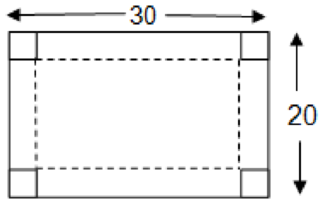
Modelling Using Cubic Functions

2.9.1: If You Build It...

Date: Mar. 8/18

1. Each member of your group should
   - cut out a 20 by 30 rectangle using grid paper,
   - cut 4 equal squares from each corner of the rectangle (one person could cut a 1 × 1 piece out of each corner; another use 2 × 2, etc.)

   NOTE: Each member of your group should have a different size of square that is being cut from the four corners,
   - fold the paper to create an open box (fold along dotted lines),
   - determine the dimensions of their box and record the information in the table.



2. Gather the data from the group and record your results in the table below.

Side Length of Square	Length of Box	Width of Box	Height of Box	Volume of Box cm^3
1	28	18	1	$1 \cdot 18 \cdot 28 =$ 504
2	26	16	2	$2 \cdot 26 \cdot 18 =$ 832
3	24	14	3	$3 \cdot 24 \cdot 14 =$ 1008
4	22	12	4	$4 \cdot 22 \cdot 12 =$ 1056
x	$30-2x$	$20-2x$	x	$x(30-2x)(20-2x)$

3. Express the volume as a function of its side length

   $V(x) = x(30-2x)(20-2x)$

4. Enter the function as Y_1 in a graphing calculator.

5. Which of the following would be the best choice for Xmin and Xmax? Justify your choices.

Xmin	Xmax	Justification
❒ -200	❒ -200	
☑ -10	❒ -10	
❒ 20	☑ 20	

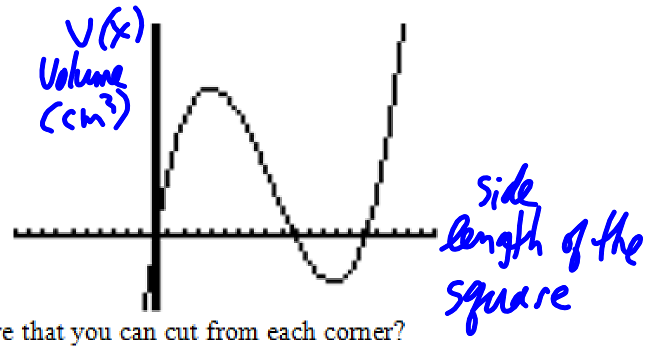
6. Which of the following would be the best choice for Ymin and Ymax? Justify your choices.

Ymin	Ymax	Justification
☑ -500	❒ -500	
❒ 10	❒ 10	
❒ 1500	☑ 1500	

7. Fill in the values for Xmin, Xmax, Ymin and Ymax into the window at right.

```
WINDOW
 Xmin=-10
 Xmax=20
 Xscl=1
 Ymin=-500
 Ymax=1500
 Yscl=100
 Xres=1
```

8. The graph produced should appear as below.
Label your axes using words and variables.



9. What limitations are there on the size of the square that you can cut from each corner?

the square must be less than 10 cm in length.

10. State the domain and range of the volume function $V(x) = x(30-2x)(20-2x)$

Domain: $\{x \in \mathbb{R} \mid 0 < x < 10\}$ Range: $\{y \in \mathbb{R} \mid 0 \leq y \leq 1056.30\}$

How are the restrictions evident from the	Domain	Range
box you are constructing?		
equation of the function?	$V(x)=0$ $20-2x$	
graph of the function?	x-int are $x=0$, $x=10$	

11. From which model in #10 do you find easiest to identify the domain and range? Support your choice.

the graph.

12. The volume function from #10 is expressed in **function notation**.
In this scenario, V(2.5) means to determine the volume of the box when $x = 2.5$.
This is done by substituting 2.5 for every occurrence of x.
Use your function in question #10 to evaluate the function when $x = 2.5$. Show all of your work.

$V(2.5) = x(30-2x)(20-2x)$
$= (2.5)(30-2(2.5))(20-2(2.5))$
$= 2.5(30-5)(20-5)$
$= 2.5(25)(15)$
$= 937.5\ cm^3$

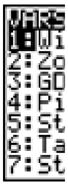
13. Using the "Trace" feature:

a) What is the value of the function when $x = 2.5$? 937.5 cm^3

b) What do both of these values represent in the context of the box?

if you cut a square 2.5 cm in length, the volume will be 937.5 cm^3.

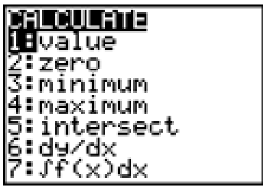
13. Follo



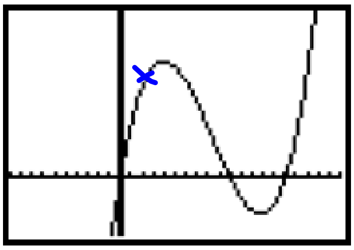
a) What

b) What

14. Graph the function and use the VALUE option in the CALC menu to determine the value of the function when $x = 2.5$.



How is this represented on the graph of the function?



15a. Complete the following table.

Side Length	Point on Graph	Value of function V(x) =
$x = 0.5$	(0.5, 275.5)	$V(0.5) = (0.5)(30-2(0.5))(20-2(0.5))$ $(0.5)(29)(19)$ $= 275.5$
$x = 4.5$	(4.5, 1039.5)	
$x = 2.7$	(2.7, 969.732)	

15b. Compare the points on the graph with the values of the volume found using substitution.

they are the same.

16. What are the dimensions that maximize the volume?

Maximum Volume	1056.3
Height of box at maximum	3.92
Length of box at maximum	22.15
Width of box at maximum	12.15

17. Using the graph, how do you know this is the maximum value?

Used the software on the calculator.

18. How could you use the Volume function for V(x) to show that the maximum occurs at the point you found?

19. If the initial dimensions of the box were doubled (i.e. 40 by 60), what do you think the maximum volume would be and the height of the box that gives that maximum?
Support your work by using a function and a graph.