MCF 3MI

4.4 Investigating the Nature of the Roots

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Recall: The quadratic formula allows us to find the roots, x, of a quadratic equation, $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The expression b^2-4ac is the part under the square root sign and is called the "discriminant". It allows us to determine the "nature of the roots' (number of roots and the type of root).

$b^2-4ac>0 \Rightarrow$ *two distinct real solutions* (roots) (and 2 x-intercepts)

$b^2-4ac=0 \Rightarrow$ *one real solution* (root) (and 1 x-intercept)

$b^2-4ac<0 \Rightarrow$ *no real solution* (roots) (and no x-intercepts)

Ex. 1: Write the discriminant. Do not evaluate.

a) $x^2+11x=6x^2-17$

$x^2+11x-6x^2+17=0$

$-5x^2+11x+17=0$

$a=-5 \quad b=11 \quad c=17$

b^2-4ac

$=(11)^2-4(-5)(17)$

b^2-4ac

$=(11)^2-4(-5)(17)$

$=121+340$

$=461$

b) $(x+3)(2x-1)=4(x+2)$

$2x^2-x+6x-3=4x+8$

$2x^2+5x-3-4x-8=0$

$2x^2+x-11=0$

$a=2 \quad b=1 \quad c=-11$

b^2-4ac

$=(1)^2-4(2)(-11)$

b^2-4ac

$=(1)^2-4(2)(-11)$

$=89$

Ex. 2: Determine the number of solutions of each quadratic equation. Do not solve.

a) $2x^2+4x-9=0$

b^2-4ac

$=(4)^2-4(2)(-9)$

$=88$

$\because b^2-4ac>0$

$\therefore 2$ solutions

b) $2x^2-3x+8=0$

b^2-4ac

$=(-3)^2-4(2)(8)$

$=-55$

$\because b^2-4ac<0$

$\therefore$ no real solutions

c) $4x^2-8x=4x-9$

$4x^2-8x-4x+9=0$

$4x^2-12x+9=0$

$a=4 \quad b=-12 \quad c=9$

b^2-4ac

$=(-12)^2-4(4)(9)$

$=144-144$

$=0$

$\because b^2-4ac=0$

$\therefore 1$ solution

Ex. 3: For what value of m does $g(x)=49x^2-28x+m$ have no zeros?

$a=49 \quad b=-28 \quad c=m$

$b^2-4ac<0$

$(-28)^2-4(49)(m)<0$

$784-196m<0$

$\frac{784}{196}<\frac{196m}{196}$

$4<m$

$m>4$

$-196m<-784$

$m>\frac{-784}{-196}$

$m>4$

$\therefore$ if $m>4$, $g(x)$ has no zeros.

$2x = 8$

$x = 4$

$-2x < -8$ $\quad$ $2x < -8$ $\quad$ $-2x < 8$

$x > \frac{-8}{-2}$ $\quad$ $x < \frac{-8}{2}$ $\quad$ $x > \frac{8}{-2}$