

MCF 3MI **4.4 Investigating the Nature of the Roots**

Date: *Mar. 29/18*

Recall: The quadratic formula allows us to find the roots, x , of a quadratic equation, $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression $b^2 - 4ac$ is the part under the square root sign and is called the “discriminant”. It allows us to determine the “nature of the roots” (number of roots and the type of root).

$b^2 - 4ac > 0 \Rightarrow$ *two distinct real solutions* (roots) (and 2 x -intercepts)

$b^2 - 4ac = 0 \Rightarrow$ *one real solution* (root) (and 1 x -intercept)

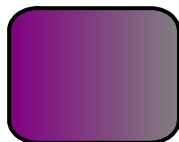
$b^2 - 4ac < 0 \Rightarrow$ *no real solution* (roots) (and no x -intercepts)

Ex. 1: Write the discriminant. Do not evaluate.

a) $x^2 + 11x = 6x^2 - 17$

*$x^2 + 11x - 6x^2 + 17 = 0$
 $-5x^2 + 11x + 17 = 0$
 $a = -5 \quad b = 11 \quad c = 17$
 $b^2 - 4ac$
 $= (11)^2 - 4(-5)(17)$*

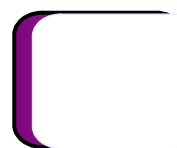
$b^2 - 4ac$
 $= (11)^2 - 4(-5)(17)$
 $= 121 + 340$
 $= 461$



b) $(x+3)(2x-1) = 4(x+2)$

*$2x^2 - x + 6x - 3 = 4x + 8$
 $2x^2 + 5x - 3 - 4x - 8 = 0$
 $2x^2 + x - 11 = 0$
 $a = 2 \quad b = 1 \quad c = -11$
 $b^2 - 4ac$
 $= (1)^2 - 4(2)(-11)$*

$b^2 - 4ac$
 $= (1)^2 - 4(2)(-11)$
 $= 89$



Ex. 2: Determine the number of solutions of each quadratic equation. Do not solve.

a) $2x^2 + 4x - 9 = 0$

$b^2 - 4ac$
 $= (4)^2 - 4(2)(-9)$
 $= 88$
 $\therefore b^2 - 4ac > 0$
 $\therefore 2 \text{ solutions}$

b) $2x^2 - 3x + 8 = 0$

$b^2 - 4ac$
 $= (-3)^2 - 4(2)(8)$
 $= -55$
 $\therefore b^2 - 4ac < 0$
 $\therefore \text{no real solutions}$

c) $4x^2 - 8x = 4x - 9$

$4x^2 - 8x - 4x + 9 = 0$
 $4x^2 - 12x + 9 = 0$
 $a = 4 \quad b = -12 \quad c = 9$
 $b^2 - 4ac$
 $= (-12)^2 - 4(4)(9)$
 $= 144 - 144$
 $= 0$
 $\therefore b^2 - 4ac = 0$
 $\therefore 1 \text{ solution}$

Ex. 3: For what value of m does $g(x) = 49x^2 - 28x + m$ have no zeros?

$a = 49 \quad b = -28 \quad c = m$

$b^2 - 4ac < 0$
 $(-28)^2 - 4(49)(m) < 0$

$784 - 196m < 0$

$\frac{784}{196} < \frac{196m}{196}$

$4 < m$

$m > 4$

$-196m < -784$

$m > \frac{-784}{-196}$

$m > 4$

\therefore if $m > 4$, $g(x)$ has no zeros.

$$2x = 8$$

$$x = 4$$

$$-2x < -8$$

$$2x < -8$$

$$-2x < 8$$

$$x > \frac{8}{2}$$

$$x < \frac{8}{2}$$

$$x > \frac{8}{2}$$