Before we begin, are there any questions from last day's work? 5.1.1

16

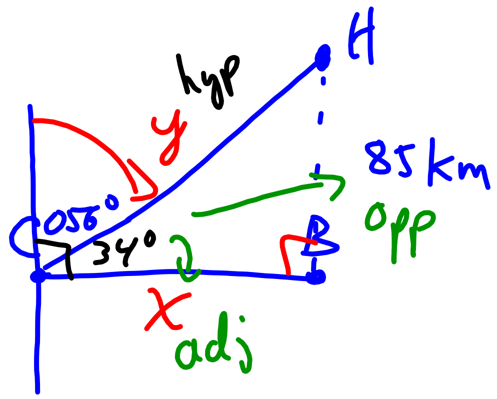
# Today's Learning Goal(s):

By the end of the class, I will be able to:

a) Correctly write the sine **LAW** and the cosine **LAW** in one of the two forms.
b) Use the sine law and cosine law to solve a non-right triangles.

16. A helicopter is often used for emergency medical transport.
It flies from its base B, due west to a small community C. It picks up two patients and a medical attendant.
The helicopter then flies on a bearing of 056° to a hospital H.
The hospital is 85 km due north of the helicopter base.
Determine how far the helicopter travels on each leg of its journey.

a) from B to C $x$    b) from C to H $y$




$$\tan 34^\circ = \frac{85}{x}$$

$$x = \frac{85}{\tan 34^\circ}$$

$$\doteq 126.0176$$

$$\doteq 126.018 \text{ km}$$

$$\sin 34^\circ = \frac{85}{y}$$

$$y = \frac{85}{\sin 34^\circ}$$

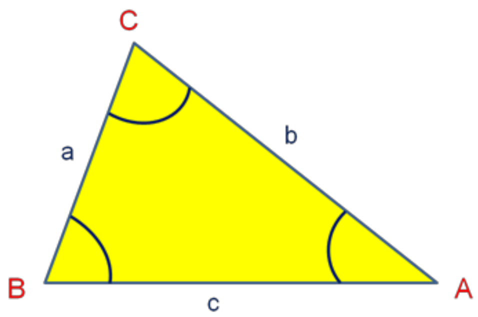
$$\doteq 152.0047$$

$$\doteq 152.005 \text{ km}$$

## 5.2.1 Reviewing the Sine **LAW** and Cosine **LAW** (to Solve Oblique Triangles)

Date: Apr. 25/18

The Sine Law can be used with any triangle, even if it is not a right triangle.
Given any triangle,




① $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

and

② $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{A}{C} = \frac{B}{D}$$

$AD = BC$

If you are trying to determine an unknown side, then use the formula given in ①.
If you are trying to determine an unknown angle, then use the formula given in ②

### So why do we need the Cosine Law?

When the triangle we are solving involves 2 known sides and the contained angle (a.k.a. SAS), then we use the formula given in ③. Remember to take the square root of the answer to find $a$.

③ $$a^2 = b^2 + c^2 - 2bc\cos A$$ (SAS)

When the triangle we are solving involves 3 known sides, but no known angles (a.k.a. SSS), then we use the formula given in ④. Remember to take the inverse cos, (or $\cos^{-1}$) to find the measure of angle A.
Note: In this case, always find the largest angle first, in case it is an obtuse angle.
The largest angle will be located opposite the longest side. [Think about it!]

④ $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$ (SSS)

Ex. 1 Solve the triangle. (Round side lengths to 3 decimal places and angles to 2 decimal places.)

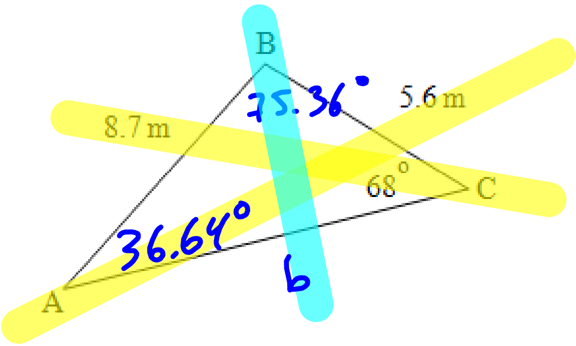



*Diagram is not drawn to scale.*

| $\angle A$ | $\angle B$ | b |
|---|---|---|
| $\frac{\sin A}{5.6} = \frac{\sin 68°}{8.7}$<br>$\frac{\cancel{8.7}\sin A}{\cancel{8.7}} = \frac{5.6\sin 68°}{8.7}$<br>$\sin A = 5.6 \times \frac{\sin 68°}{8.7}$ | $B \doteq 180° - 68° - 36.64°$<br>$\doteq 75.36°$ | $\frac{b}{\sin 75.36°} \doteq \frac{8.7}{\sin 68°}$<br>$b \doteq \sin 75.36° \times \frac{8.7}{\sin 68°}$<br>$\doteq 9.0786$<br>$\doteq 9.079$ m |

$$A = \sin^{-1}\left(5.6 \times \frac{\sin 68°}{8.7}\right)$$

$\doteq 36.641$

$\doteq 36.64°$

Ex. 2 Solve the triangle. (Round according to our rules.)

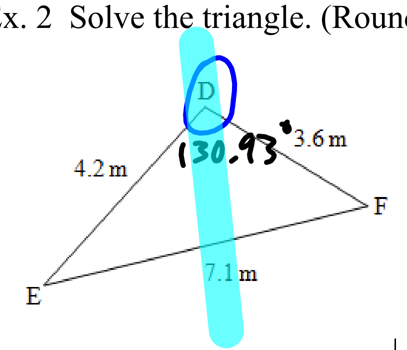



*Diagram is not drawn to scale.*

$\angle D$

Since we have SSS, use ④

$$\cos D = \frac{4.2^2 + 3.6^2 - 7.1^2}{2(4.2)(3.6)}$$

$$D = \cos^{-1}\left(\frac{4.2^2 + 3.6^2 - 7.1^2}{2 \times 4.2 \times 3.6}\right)$$

$$\doteq \cos^{-1}\left(\frac{-19.81}{30.24}\right)$$

$$\doteq \cos^{-1}(-0.655)$$

$$\doteq 130.926$$

$$\doteq 130.93^\circ$$

$\angle F$

Now use the sine law

$$\frac{\sin F}{4.2} \doteq \frac{\sin 130.93^\circ}{7.1}$$

$$\sin F \doteq \frac{4.2 \sin 130.93^\circ}{7.1}$$

$$F \doteq \sin^{-1}\left(\frac{4.2 \sin 130.93^\circ}{7.1}\right)$$

$$\doteq 26.546$$

$$\doteq 26.55^\circ$$

$\angle E$

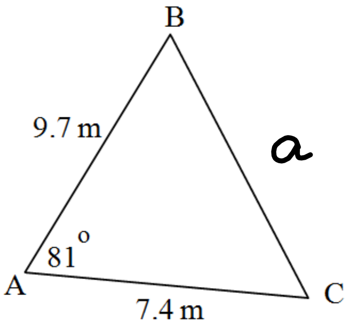
Now use the triangle sum

$$\angle E \doteq 180^\circ - 130.93^\circ - 26.55^\circ$$

$$\doteq 22.52^\circ$$

Ex. 3 Solve the triangle. (Round according to our rules.)

Diagram is not drawn to scale.




| $a$ | $\angle B$ | $\angle C$ |
|---|---|---|
| Since we have SAS, use ③ | Now use the sine law (it's easier) | Now use the triangle sum |
| $a^2 = 9.7^2 + 7.4^2 - 2(9.7)(7.4)\cos 81°$ | $\frac{\sin B}{b} = \frac{\sin A}{a}$ | $\angle C \doteq 180° - 81° - 40.55°$ |
| $\doteq 126.392$ this value is $a^2$ | $\frac{\sin B}{7.4} \doteq \frac{\sin 81°}{11.242}$ | $\doteq 58.45°$ |
| $a \doteq \sqrt{126.392}$ | $\sin B \doteq \frac{7.4 \sin 81°}{11.242}$ | |
| $\doteq 11.2424$ | $B \doteq \sin^{-1}\left(\frac{7.4 \sin 81°}{11.242}\right)$ | |
| $\doteq 11.242$ m | $\doteq 40.552$ | |
| | $\doteq 40.55°$ | |