Date: ____________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) prove trigonometric identities.

9

4a c

2ac 11

Last day's work: pp. 338-339 #1 – 5, 8 – 13
p. 340 #2

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2. Determine the exact value of each trigonometric expression. Express your answers in simplified radical form.

a) $(\sin 45°)(\cos 45°) + (\sin 30°)(\cos 60°)$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{2}{4} + \frac{1}{4}$

$= \frac{3}{4}$

c) $\tan 30° + 2(\sin 45°)(\cos 60°)$

$= \frac{1}{\sqrt{3}} + 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{3} + \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{3}}{6} + \frac{3\sqrt{2}}{6}$

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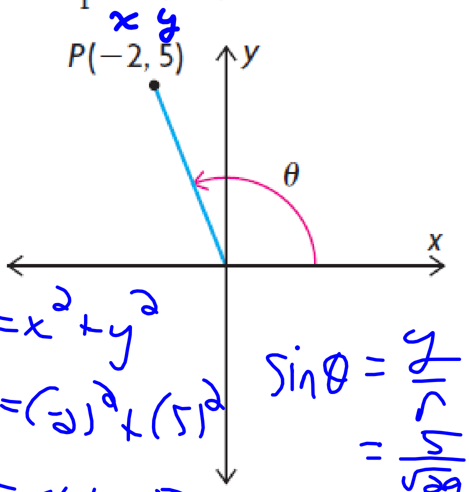
4. For each sketch, state the primary trigonometric ratios associated with angle θ. Express your answers in simplified radical form.

a)



$r^2 = x^2 + y^2$

$= (-2)^2 + (5)^2$

$= 4 + 25$

$= 29$

$\therefore r = \sqrt{29}$

$\sin\theta = \frac{y}{r}$

$= \frac{5}{\sqrt{29}}$

$\cos\theta = \frac{x}{r}$

$= \frac{-2}{\sqrt{29}}$

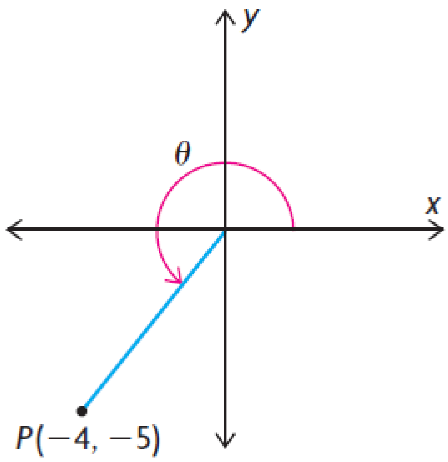
$\tan\theta = \frac{y}{x}$

$= \frac{5}{-2}$

$\sec\theta$

$= \frac{-\sqrt{29}}{2}$

c)



$r^2 = 16 + 25$

$r = \sqrt{41}$

$\sin\theta = \frac{y}{r}$

$= \frac{-5}{\sqrt{41}}$

$\cos\theta = \frac{x}{r}$

$= \frac{-4}{\sqrt{41}}$

$\tan\theta = \frac{y}{x}$

$= \frac{-5}{-4}$

$= \frac{5}{4}$

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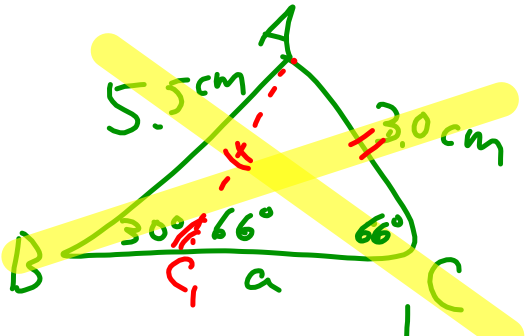
8. Determine whether it is possible to draw a triangle given each set of information. Sketch all possible triangles where appropriate. Calculate, then label, all side lengths to the nearest tenth of a centimetre and all angles to the nearest degree.

a) $b = 3.0$ cm, $c = 5.5$ cm, $\angle B = 30°$

b) $b = 12.2$ cm, $c = 8.2$ cm, $\angle C = 34°$

c) $a = 11.1$ cm, $c = 5.2$ cm, $\angle C = 33°$



$\angle C$	$\angle A$	a
$\frac{\sin C}{5.5} = \frac{\sin 30°}{3.0}$ $C \doteq 66°$	$A \doteq 180° - 30° - 66°$ $\doteq 84°$	$\frac{a}{\sin 84°} = \frac{3.0}{\sin 30°}$ $a \doteq 5.96$ cm $= 6.0$ cm
$C \doteq 180° - 66°$ $= 114°$	$A \doteq 180° - 30° - 114°$ $= 36°$	$\frac{a}{\sin 36°} = \frac{3.0}{\sin 30°}$ $a \doteq 3.52$ $= 3.5$ cm

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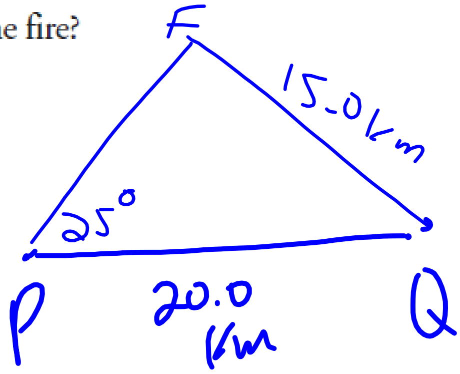
9. Two forest fire stations, P and Q, are 20.0 km apart. A ranger at station Q sees a fire 15.0 km away. If the angle between the line PQ and the line from P to the fire is $25°$, how far, to the nearest tenth of a kilometre, is station P from the fire?



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11. Two spotlights, one blue and the other white, are placed 6.0 m apart on a track on the ceiling of a ballroom. A stationary observer standing on the ballroom floor notices that the angle of elevation is 45° to the blue spotlight and 70° to the white one. How high, to the nearest tenth of a metre, is the ceiling of the ballroom?

From the soluon manual:

11. The angle between the two angles of elevation measures 180° − (45° + 70°), or 65°. Let x by the hypotenuse of the triangle formed by the observer, the blue spotlight, and the height of the ceiling at the blue spotlight.

$$\frac{x}{\sin 70°} = \frac{6}{\sin 65°}$$

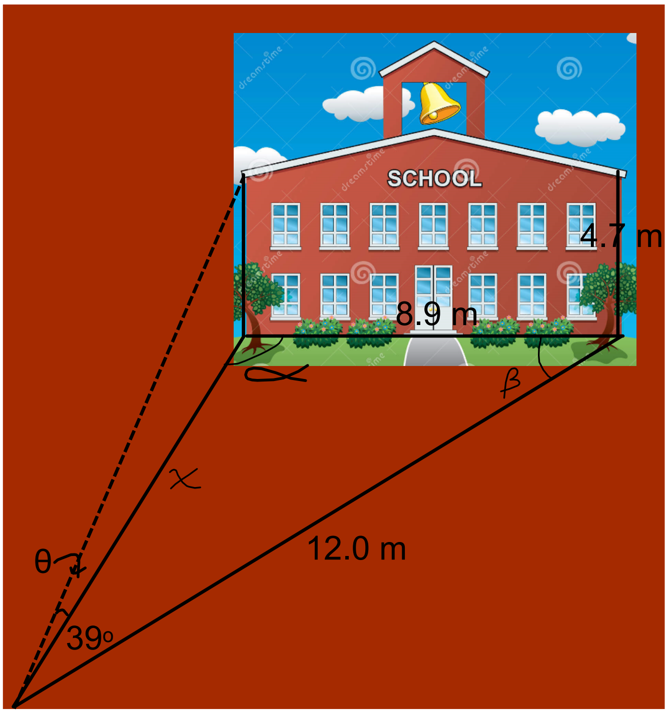
$$x \doteq 6.2$$

$$\sin 45° = \frac{h}{6.2}$$

$$h \doteq 4.4 \text{ m}$$

p. 339 **Soluon #1**

13. While standing at the left corner of the schoolyard in front of her school, Suzie estimates that the front face is 8.9 m wide and 4.7 m high. From her position, Suzie is 12.0 m from the base of the right exterior wall. She determines that the left and right exterior walls appear to be 39° apart. From her position, what is the angle of elevation, to the nearest degree, to the top of the left exterior wall?



Suzie

$$\frac{\sin\alpha}{12} = \frac{\sin 39^\circ}{8.9}$$

$$\alpha = \sin^{-1}\left(12 \times \frac{\sin 39^\circ}{8.9}\right)$$

$$\doteq 58.05^\circ$$

But α is obtuse

$$\therefore \alpha \doteq 121.9^\circ$$

$$\therefore \beta \doteq 180^\circ - 39^\circ - 121.9^\circ$$

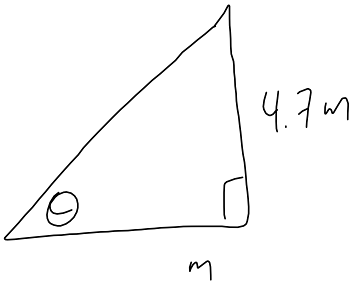
$$\doteq 19.1^\circ$$

$$\frac{x}{\sin 19.1^\circ} = \frac{8.9}{\sin 39^\circ}$$

$$x \doteq \sin 19.1^\circ \times \frac{8.9}{\sin 39^\circ}$$

$$\doteq 4.62$$

$$\doteq 4.6$$



$$\tan\theta \doteq \frac{4.7}{4.6}$$

$$\theta \doteq \tan^{-1}\left(\frac{4.7}{4.6}\right)$$

$$\doteq 45.6$$

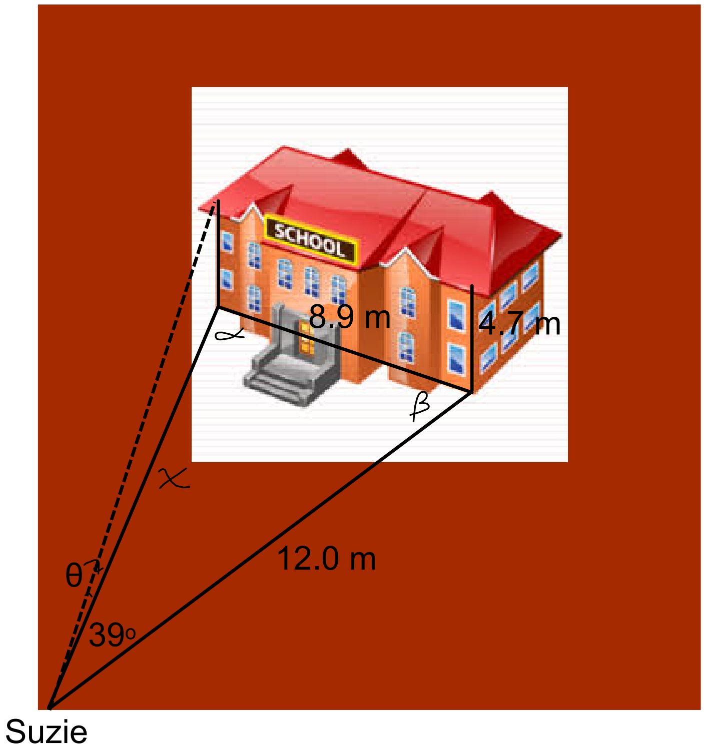
$$\doteq 46^\circ$$

$\therefore$ the angle of elevation is 46° [some texts have 46° at back.]

but see next page.

p. 339 **Soluon #2**

13. While standing at the left corner of the schoolyard in front of her school, Suzie estimates that the front face is 8.9 m wide and 4.7 m high. From her position, Suzie is 12.0 m from the base of the right exterior wall. She determines that the left and right exterior walls appear to be 39° apart. From her position, what is the angle of elevation, to the nearest degree, to the top of the left exterior wall?



$$\frac{\sin\alpha}{12} = \frac{\sin 39^\circ}{8.9}$$

$$\alpha = \sin^{-1}\left(12 \times \frac{\sin 39^\circ}{8.9}\right)$$

$$\doteq 58.05^\circ$$

$$\therefore \beta \doteq 180^\circ - 39^\circ - 58.1^\circ$$

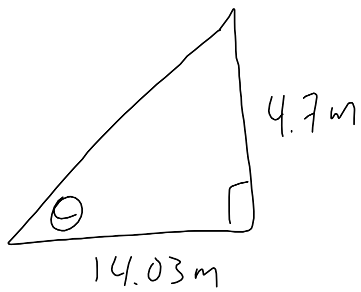
$$\doteq 82.9^\circ$$

$$\frac{x}{\sin 82.9^\circ} = \frac{8.9}{\sin 39^\circ}$$

$$x \doteq \sin 82.9^\circ \times \frac{8.9}{\sin 39^\circ}$$

$$\doteq 14.033$$

$$\doteq 14.03$$



$$\tan\theta \doteq \frac{4.7}{14.03}$$

$$\theta \doteq \tan^{-1}\left(\frac{4.7}{14.03}\right)$$

$$\doteq 18.51$$

$$\doteq 19^\circ$$

$\therefore$ the angle of elevation is 19°

[Some texts say 18° at the back?]

5.5 Trigonometric Identities

Date: May 8/18

Equations are valid for only certain values of the variable.
For example:

$2x + 1 = 7$

$2x = 7 - 1$

$2x = 6$

$x = 3$

$x = 3$ is the only value to make the equation true.

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$\therefore x = 7$ or $x = -2$

$x = 7$ and $x = -2$ are the only values to make the equation true.

Identities are valid for **all values** of the variable.
For example:

$2(x + 3) = 2x + 6$ $\qquad$ $x^2 + 6x + 9 = (x + 3)^2$

Let's start with the circle definitions to develop some identities that we can use later.

SYR CXR TYX

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

To Prove an Identity:

*** Separate the LS and RS, and work on them separately**

Ex.1 Prove that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$LS = \tan\theta$

$= \dfrac{y}{x}$

$RS = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\frac{y}{r}}{\frac{x}{r}}$

$= \dfrac{y}{r} \div \dfrac{x}{r}$

$= \dfrac{y}{\cancel{r}} \times \dfrac{\cancel{r}}{x}$

$= \dfrac{y}{x}$

$\because LS = RS$

$\therefore$ QED!

Restriction

$\cos\theta \neq 0$

$\theta \neq \cos^{-1}(0)$

$\theta \neq 90^\circ$

Q.E.D. (also written QED)

"quod erat demonstrandum"

"that which was to be demonstrated"

Ex.2 Prove that $\sin^2\theta+\cos^2\theta=1$

$LS=\sin^2\theta+\cos^2\theta$ $\qquad$ $RS=1$

$=\left(\frac{y}{r}\right)^2+\left(\frac{x}{r}\right)^2$

$=\frac{y^2}{r^2}+\frac{x^2}{r^2}$

$=\frac{y^2+x^2}{r^2}$ * But we know $x^2+y^2=r^2$ (PT)

$=\frac{r^2}{r^2}$

$=1$

$\therefore LS=RS$

$\therefore$ QED!

Ex.3 Prove that Use "known" identities; i.e. known since Ex.1&2

a) $\dfrac{\cos\alpha\tan\alpha}{\sin\alpha} = 1$

b) $\cos\phi = \dfrac{1}{\cos\phi} - \sin\phi\tan\phi$

$LS = \dfrac{\cos\alpha \tan\alpha}{\sin\alpha}$ $RS = 1$

$= \dfrac{\cancel{\cos\alpha}\left(\dfrac{\sin\alpha}{\cancel{\cos\alpha}}\right)}{\sin\alpha}$

$= \dfrac{\sin\alpha}{\sin\alpha}$

$= 1$

$\therefore LS = RS$

$\therefore$ QED.

$LS = \cos\phi$ $RS = \dfrac{1}{\cos\phi} - \sin\phi\tan\phi$

$= \dfrac{1}{\cos\phi} - \sin\phi\left(\dfrac{\sin\phi}{\cos\phi}\right)$

$= \dfrac{1}{\cos\phi} - \dfrac{\sin^2\phi}{\cos\phi}$

$= \dfrac{1-\sin^2\phi}{\cos\phi}$

$= \dfrac{\cos^2\phi}{\cos\phi}$

$= \cos\phi$

$\therefore LS = RS$

$\therefore$ QED!

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

or

$\sin^2\theta = 1 - \cos^2\theta$

$\dfrac{A^2}{A} = A^{2-1} = A$

Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 338-339 #1 – 5, 8 – 13
p. 340 #2

Study for the Unit 5 Summative!

Today's Homework Practice includes:
p. 310 #1 – 6

Work ahead? pp. 310-311 #8, 10 – 12 [14]
Worksheet a – j (***online***)

Note: Sometimes using substitution can help simplify a question.
Ex. Simplify $(1 - \cos\theta)(1 + \cos\theta)$ Change to $(1 - a)(1 + a)$

$= 1 - \cos^2\theta$ $= 1 - a^2$