Date: _____

Today's Learning Goal(s):

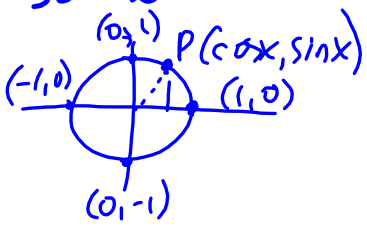
By the end of the class, I will be able to:

a) prove trigonometric identities.

Last day's work: p. 310 #1 – 6

5. Prove each identity. State any restrictions on the variables.

5a 6



a) $\dfrac{\sin x}{\tan x} = \cos x$

$LS = \dfrac{\sin x}{\tan x}$ $\quad RS = \cos x$

Restrictions

$\tan x \neq 0$ $\quad \cos x \neq 0$ $\quad \sin x \neq 0$

$x \neq 90°$ $\quad x \neq 0°$

$x \neq 270°$ $\quad x \neq 180°$

$x \neq 360°$

$= \sin x \div \tan x$

$= \sin \div \dfrac{\sin x}{\cos x}$

$= \sin x \times \dfrac{\cos x}{\sin x}$

$= \cos x$

$\therefore LS = RS$

$\therefore$ QED.

6. Mark claimed that $\dfrac{1}{\cot \theta} = \tan \theta$ is an identity. Marcia let $\theta = 30°$ and found that both sides of the equation worked out to $\frac{1}{\sqrt{3}}$. She said that this proves that the equation is an identity. Is Marcia's reasoning correct? Explain.

Marcia's reasoning is NOT correct, because she only showed that Mark is correct for 1 specific case, not for ALL cases, as is the case for a true identity.
Note: Mark's example IS an identity; it's one of the reciprocal identities.

5.5 Trigonometric Identities (Day2)

Date: May 10/18

Recall:

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \quad \begin{cases} \sin^2\theta = 1 - \cos^2\theta \\ \cos^2\theta = 1 - \sin^2\theta \end{cases}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

To Prove an Identity:

* **Separate the LS and RS, and work on them separately**
* **convert *tan* and reciprocal ratios to*sin* or *cos***
* **apply the Pythagorean Identity, use common denominators & factor as require**

Don't forget that "Math is FUN!"

Ex.1 Prove that $\dfrac{\sin^2 x}{1-\cos x} = 1+\cos x$

$LS = \dfrac{\sin^2 x}{1-\cos x}$ $RS = 1+\cos x$

$= \dfrac{1-\cos^2 x}{1-\cos x}$

$= \dfrac{(1-\cos x)(1+\cos x)}{1-\cos x}$

$= 1+\cos x$

$\therefore LS = RS$

$\therefore$ QED!

$1-x^2$

$= (1-x)(1+x)$

$\sin^2 x + \cos^2 x = 1$ (pt)

$\sin^2 x = 1-\cos^2 x$

Ex.2 Prove that $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=\frac{2}{\cos^2\theta}$

$LS = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$ $\qquad RS = \frac{2}{\cos^2\theta}$

$= \frac{1}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right) + \frac{1}{1-\sin\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right)$

$= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$

$= RS$

$\therefore LS = RS$

$\therefore$ Q.E.D.

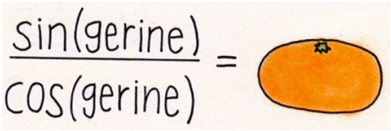
$\frac{2}{5}\times\frac{2}{2} = \frac{4}{10}$

$(1-x)(1+x) = 1-x^2$

Are there any Homework Questions you would like to see on the board?

Last day's work: p. 310 #1 – 6

Laugh or Groan?



Today's Homework Practice includes:

pp. 310-311 #8, 10 – 12 [14]

Worksheet a – j (*online*)

8. Prove each identity. State any restrictions on the variables.

a) $\dfrac{\sin^2\phi}{1-\cos\phi} = 1+\cos\phi$

b) $\dfrac{\tan^2\alpha}{1+\tan^2\alpha} = \sin^2\alpha$

c) $\cos^2 x = (1-\sin x)(1+\sin x)$

d) $\sin^2\theta + 2\cos^2\theta - 1 = \cos^2\theta$

e) $\sin^4\alpha - \cos^4\alpha = \sin^2\alpha - \cos^2\alpha$

f) $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

b) Solution 1: LS - use the quotient identity, and then simplify the fraction.
Solution 2: LS - use version 2 of the Pythagorean identity.

d) LS - sub in $\sin^2\theta$

e) LS - factor the difference of squares

f) LS - add the fractions then sub for $\tan\theta$

12. Prove each identity. State any restrictions on the variables.

T

a) $\dfrac{\sin^2\theta + 2\cos\theta - 1}{\sin^2\theta + 3\cos\theta - 3} = \dfrac{\cos^2\theta + \cos\theta}{-\sin^2\theta}$

b) $\sin^2\alpha - \cos^2\alpha - \tan^2\alpha = \dfrac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1-\sin^2\alpha}$

a) sub for $\sin^2\theta$ on both sides, then factor and divide.

b) LS - sub for $\sin^2\theta$ and use the quotient rule, then add the fractions.

Extending

14. **a)** Which equations are not identities? Justify your answers.
b) For those equations that are identities, state any restrictions on the variables.

i) $(1-\cos^2 x)(1-\tan^2 x) = \dfrac{\sin^2 x - 2\sin^4 x}{1-\sin^2 x}$

ii) $1 - 2\cos^2\phi = \sin^4\phi - \cos^4\phi$

iii) $\dfrac{\sin\theta\tan\theta}{\sin\theta+\tan\theta} = \sin\theta\tan\theta$

iv) $\dfrac{1+2\sin\beta\cos\beta}{\sin\beta+\cos\beta} = \sin\beta+\cos\beta$

v) $\dfrac{1-\cos\beta}{\sin\beta} = \dfrac{\sin\beta}{1+\cos\beta}$

vi) $\dfrac{\sin x}{1+\cos x} = \csc x - \cot x$

iv) LS - sub $\sin^2\theta + \cos^2\theta$ in for 1, and then factor and divide.

v) LS - multiply top and bottom by $1+\cos\beta$

vi) RS - put in terms of sinx and cosx and then see above.