

Student Solutions to Homework

p. 310 #8b

$$\text{b) } \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$$

8b) $\frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$
 $\text{LS} = \frac{\sin^2 \alpha}{\cos^2 \alpha}$ $\therefore \text{LS} = \text{RS}$
 $\text{Sec}^2 \alpha$ $\therefore \text{QED}$
 $= \frac{\sin^2 \alpha}{\cos^2 \alpha} \div \frac{1}{\cos^2 \alpha}$
 $= \frac{\sin^2 \alpha}{\cos^2 \alpha} \times \frac{\cos^2 \alpha}{1}$
 $= \sin^2 \alpha$

8.6) $\frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$
 $\text{LS} = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha}$ $\text{RS} = \sin^2 \alpha$
 $= \frac{\sin^2 \alpha}{\cos^2 \alpha} \div \frac{1}{\cos^2 \alpha}$
 $= \frac{\sin^2 \alpha}{\cos^2 \alpha} \times \frac{\cos^2 \alpha}{1}$
 $= \sin^2 \alpha$

#8 b) $\frac{\tan^2 \theta}{1 + \tan^2 \theta} = \sin^2 \theta$

$\frac{\text{LS}}{\text{RS}}$

$$= \frac{\left(\frac{\sin \theta}{\cos \theta} \right)^2}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}$$

$$= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}$$

$$= \frac{\sin^2 \theta}{\sin^2 \theta + \cos^2 \theta}$$

$$= \frac{\sin^2 \theta}{1}$$

$$= \sin^2 \theta$$

p. 311

10. Is $\csc^2 \theta + \sec^2 \theta = 1$ an identity? Prove that it is true or demonstrate why it is false.

$$\begin{aligned}
 & \text{LHS} = (\csc^2 \theta + \sec^2 \theta) \\
 &= \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \quad \text{RHS} = 1 \\
 &= \frac{1(\cos^2 \theta) + 1(\sin^2 \theta)}{\sin^2 \theta (\cos^2 \theta)} \\
 &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta (\cos^2 \theta)} \\
 &= \frac{1}{\sin^2 \theta (\cos^2 \theta)} \\
 &\because \text{LHS} \neq \text{RHS} \quad \therefore \text{not an identity}
 \end{aligned}$$

$$\begin{aligned}
 & \text{LHS} = \csc^2 \theta + \sec^2 \theta \\
 &= \csc^2(45^\circ) + \sec^2(45^\circ) \\
 &= 4 \\
 &\therefore \text{LHS} \neq \text{RHS} \\
 &\therefore \text{The equation is not an identity}
 \end{aligned}$$

p. 311 #12

Looks like I did not get pics of the final proofs for 12b.

My apologies :(

b) $\sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha = \frac{2 \sin^2 \alpha - 2 \sin^4 \alpha - 1}{1 - \sin^2 \alpha}$

12b)

$$\begin{aligned} \sin^2 \theta - \cos^2 \theta - \tan^2 \theta &= 2 \sin^2 \theta - 2 \sin^4 \theta - 1 \\ L.S. &= \sin^2 \theta - \cos^2 \theta - \tan^2 \theta \\ &= \sin^2 \theta - (\cos^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta}) \\ &= \sin^2 \theta - (1 - \sin^2 \theta) - \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \sin^2 \theta - 1 + \sin^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= 2 \sin^2 \theta - 1 - \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= 2 \sin^2 \theta (\cos^2 \theta) - (\cos^2 \theta) - \sin^2 \theta (\cos^2 \theta) \\ &= 2 \sin^2 \theta (\cos^2 \theta) - 1(1 - \sin^2 \theta) - \sin^2 \theta (1 - \sin^2 \theta) \\ &= 2 \sin^2 \theta - 2 \sin^4 \theta - 1 + \sin^2 \theta + \sin^2 \theta + \sin^4 \theta \\ &= \end{aligned}$$

LS

$$\begin{aligned} \sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha &= \sin^2 \alpha - \cos^2 \alpha - \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{\sin^2 \alpha - \cos^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{\sin^2 \alpha - (\cos^2 \alpha + \sin^2 \alpha)}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{-\sin^4 \alpha - (-2 \sin^2 \alpha + \sin^4 \alpha)}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{-\sin^4 \alpha - 1 + 2 \sin^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{2 \sin^2 \alpha - 1 + 2 \sin^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} - \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{2 \sin^2 \alpha \times \cos^2 \alpha - \cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{\sin^2 \alpha (1 - \sin^2 \alpha) - (1 - \sin^2 \alpha)^2 - \sin^2 \alpha}{\cos^2 \alpha} \\ &= \frac{-\sin^4 \alpha - (1 - 2 \sin^2 \alpha + \sin^4 \alpha)}{\cos^2 \alpha} \\ &= \end{aligned}$$

RS

$$\begin{aligned} 2 \sin^2 \theta - 2 \sin^4 \theta - 1 &= \frac{2 \sin^2 \theta - 2 \sin^4 \theta - 1}{1 - \sin^2 \theta} \\ 2(1 - \cos^2 \theta) - 2(\sin^2 \theta)(\sin^2 \theta) &= \\ (1 - \cos^2 \theta) &= \\ 2 \sin^2 \theta - 2(\sin^4 \theta) - 1 &= \\ \frac{\cos^2 \theta}{2 \sin^2 \theta - 2(\sin^4 \theta)} &= \end{aligned}$$

Worksheet b) $\cot^2 a = \cos^2 a + (\cot a \cdot \cos a)^2$

$$\begin{aligned}
 b) & \text{ LS} & \text{ RS} \\
 & \cot^2 a & \\
 & \cos^2 a + (\cot a \cdot \cos a)^2 & \\
 & = \cos^2 a + \left(\frac{\cos a}{\sin a} \cdot \cos a \right)^2 & \\
 & = \cos^2 a + \left(\frac{\cos^2 a}{\sin^2 a} \cdot \cos^2 a \right) & \\
 & = \cos^2 a + \left(\frac{\cos^2 a \cdot \cos^2 a}{\sin^2 a} \right) & \\
 & = \cos^2 a + \frac{\cos^4 a}{\sin^2 a} & \\
 & = \frac{\cos^2 a \cdot \sin^2 a + \cos^4 a}{\sin^2 a} & \\
 & = \frac{\cos^2 a \cdot (\sin^2 a)}{\sin^2 a} + \frac{\cos^4 a}{\sin^2 a} &
 \end{aligned}$$

$$\begin{aligned}
 b) & \cot^2 a = \cos^2 a + (\cot a \cdot \cos a)^2 & \\
 & \text{LS: } \cot^2 a & \text{RS: } \cos^2 a + (\cot a \cdot \cos a)^2 \\
 & & = \cos^2 a + \left(\frac{\cos a}{\sin a} \cdot \cos a \right)^2 \\
 & & = \cos^2 a + \left(\frac{\cos^2 a}{\sin a} \right) \left(\frac{\cos^2 a}{\sin a} \right) \\
 & & = \frac{\cos^2 a}{\sin a} + \frac{\cos^4 a}{\sin^2 a} \\
 & & = \frac{\cos^2 a \cdot \sin^2 a + \cos^4 a}{\sin^2 a} \\
 & & = \frac{\cos^2 a \cdot (\sin^2 a)}{\sin^2 a} + \frac{\cos^4 a}{\sin^2 a}
 \end{aligned}$$

Date: May 11, 2018

Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) prove trigonometric identities.

$\frac{8}{10}$
 $\frac{12}{12}$

Last day's work: pp. 310-311 #8, 10 – 12 [14]
 Worksheet a – j (*online*)

p. 310 #8b

$$\text{b) } \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$$

$$\text{L.S.} = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$= \frac{\sin^2 \alpha}{\cos^2 \alpha} \div (1 + \tan^2 \alpha)$$

$$= \frac{\sin^2 \alpha}{\cos^2 \alpha} \div \sec^2 \alpha$$

$$= \frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \frac{1}{\cos^2 \alpha}$$

$$= \frac{\sin^2 \alpha}{\cos^2 \alpha} \times \frac{\cancel{\cos^2 \alpha}}{1}$$

$$= \sin^2 \alpha$$

$$= \text{R.S.} \quad \therefore \text{L.S.} = \text{R.S.}$$

$\therefore \text{Q.E.D.}$

p. 311

10. Is $\csc^2 \theta + \sec^2 \theta = 1$ an identity? Prove that it is true or demonstrate why it is false.

10. For example: $\csc^2 \theta + \sec^2 \theta = 1$ is not an identity; $\csc^2 45^\circ + \sec^2 45^\circ = 4$ shows that it is false.

p. 311 #12

$$\text{b)} \quad \sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha = \frac{2 \sin^2 \alpha - 2 \sin^4 \alpha - 1}{1 - \sin^2 \alpha}$$

$$LS = \sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha \quad RS = \frac{2 \sin^2 \alpha - 2 \sin^4 \alpha - 1}{1 - \sin^2 \alpha}$$

$$= 1 - \cos^2 \alpha - (\cos^2 \alpha - \tan^2 \alpha)$$

$$= 1 - 2\cos^2 \alpha - \frac{\sin^2 \alpha}{\cos^2 \alpha}$$

$$= \frac{\cos^2 \alpha - 2\cos^2 \alpha \left(\frac{\cos^2 \alpha}{\cos^2 \alpha} \right) - \frac{\sin^2 \alpha}{\cos^2 \alpha}}{\cos^2 \alpha}$$

$$= \frac{\cos^2 \alpha - 2\cos^4 \alpha - \sin^2 \alpha}{\cos^2 \alpha}$$

$$= \frac{1 - \sin^2 \alpha - 2(1 - \sin^2 \alpha)(1 - \sin^2 \alpha) - \sin^2 \alpha}{1 - \sin^2 \alpha}$$

$$= \frac{1 - \sin^2 \alpha - 2(1 - 2\sin^2 \alpha + \sin^4 \alpha) - \sin^2 \alpha}{1 - \sin^2 \alpha}$$

$$= \frac{1 - \sin^2 \alpha - 2 + 4\sin^2 \alpha - 2\sin^4 \alpha - \sin^2 \alpha}{1 - \sin^2 \alpha}$$

$$= \frac{-2\sin^2 \alpha + 4\sin^2 \alpha - 1 - 2\sin^4 \alpha}{1 - \sin^2 \alpha}$$

$$= \frac{2\sin^2 \alpha - 2\sin^4 \alpha - 1}{1 - \sin^2 \alpha}$$

$$= RS. \quad \therefore LS = RS$$

∴ QED

Worksheet

b) $\cot^2 a = \cos^2 a + (\cot a \cdot \cos a)^2$

$$\begin{aligned}
 RS &= \cos^2 a + (\cot a \cdot \cos a)^2 & LS &= \cot^2 a \\
 &= \cos^2 a + \cot^2 a \cdot \cos^2 a & & \\
 &= \cos^2 a (1 + \cot^2 a) & \therefore LS = RS \\
 &= \cos^2 a (\csc^2 a) & \therefore Q.E.D. \\
 &= \cos^2 a \left(\frac{1}{\sin^2 a} \right) \\
 &= \frac{\cos^2 a}{\sin^2 a} \\
 &= \cot^2 a
 \end{aligned}$$

5.5 Trigonometric Identities (Day3)Date: May 11, 2018**Recall:****Reciprocal Identities**

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

Study for the Trig Identities Quiz!

Today's Homework Practice includes:

p. 339 #6, 7

p. 340 #4

8. Prove each identity. State any restrictions on the variables.

a) $\frac{\sin^2 \phi}{1 - \cos \phi} = 1 + \cos \phi$

b) $\frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$

c) $\cos^2 x = (1 - \sin x)(1 + \sin x)$

d) $\sin^2 \theta + 2 \cos^2 \theta - 1 = \cos^2 \theta$

e) $\sin^4 \alpha - \cos^4 \alpha = \sin^2 \alpha - \cos^2 \alpha$

f) $\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta}$

b) Solution 1: LS - use the quotient identity,
and then simplify the fraction.

Solution 2: LS - use version 2 of the Pythagorean identity.

d) LS - sub in $\sin^2 \theta$

e) LS - factor the difference of squares

f) LS - add the fractions then sub for $\tan \theta$

12. Prove each identity. State any restrictions on the variables.

T a) $\frac{\sin^2 \theta + 2 \cos \theta - 1}{\sin^2 \theta + 3 \cos \theta - 3} = \frac{\cos^2 \theta + \cos \theta}{-\sin^2 \theta}$

b) $\sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha = \frac{2 \sin^2 \alpha - 2 \sin^4 \alpha - 1}{1 - \sin^2 \alpha}$

a) sub for $\sin^2 \theta$ on both sides,
then factor and divide.

b) LS - sub for $\sin^2 \theta$ and use the quotient rule,
then add the fractions.

Extending

14. a) Which equations are not identities? Justify your answers.

b) For those equations that are identities, state any restrictions on the variables.

i) $(1 - \cos^2 x)(1 - \tan^2 x) = \frac{\sin^2 x - 2 \sin^4 x}{1 - \sin^2 x}$

ii) $1 - 2 \cos^2 \phi = \sin^4 \phi - \cos^4 \phi$

iii) $\frac{\sin \theta \tan \theta}{\sin \theta + \tan \theta} = \sin \theta \tan \theta$

iv) $\frac{1 + 2 \sin \beta \cos \beta}{\sin \beta + \cos \beta} = \sin \beta + \cos \beta$

v) $\frac{1 - \cos \beta}{\sin \beta} = \frac{\sin \beta}{1 + \cos \beta}$

vi) $\frac{\sin x}{1 + \cos x} = \csc x - \cot x$

iv) LS - sub $\sin^2 \theta + \cos^2 \theta$ in for 1,
and then factor and divide.

v) LS - multiply top and bottom by $1 + \cos \beta$

vi) RS - put in terms of $\sin x$ and $\cos x$ and then see above.