

Are there any Homework Questions you would like to see on the board?

p. 415 # 1, 2cef, 3, 6, 7, 9 – 12, 14, 15
 p. 419 # 1 – 8

9a

Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) Examine the features of exponential functions and compare them with the graphs of linear and quadratic functions.
- b) Graph an exponential function (growth and decay).

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9. Write each expression using powers, then simplify. Evaluate your simplified expression.

$$\begin{aligned}
 \text{a) } & \frac{\sqrt{200}}{\sqrt{2}} \\
 &= \frac{(200)^{\frac{1}{2}}}{(2)^{\frac{1}{2}}} \\
 &= \left(\frac{200}{2}\right)^{\frac{1}{2}} \\
 &= 100^{\frac{1}{2}} \\
 &= \sqrt{100} \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{a) } & \frac{\sqrt{200}}{\sqrt{2}} \\
 &= \sqrt{200} \div \sqrt{2} \\
 &= \sqrt{200 \div 2} \\
 &= \sqrt{100}
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{aligned} & b^{\frac{m}{n}} \\ & = \left(\sqrt[n]{b}\right)^m \end{aligned} \right\} \begin{aligned} & 9^{\frac{1}{2}} \\ & = \sqrt{9} \\ & = 3 \end{aligned} \\
 & \left. \begin{aligned} & 2^5 \\ & = 2^3 \times 2^2 \end{aligned} \right\} \\
 & \left. \begin{aligned} & a^{m \cdot n} \\ & = (a^m)^n \end{aligned} \right\} \begin{aligned} & \frac{a^m}{b^n} \\ & = \left(\frac{a}{b}\right)^m \end{aligned}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{a} \times \sqrt{b} & \left| \begin{aligned} & \sqrt{4} \times \sqrt{9} \\ & = \sqrt{4 \times 9} \\ & = \sqrt{36} \\ & = 6 \end{aligned} \right. \\
 &= \sqrt{a \times b}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{x} + \sqrt{x} & \left\{ \begin{aligned} & y + y \\ & = 2y \end{aligned} \right. \left| \begin{aligned} & 2\sqrt{3} + 5\sqrt{3} \\ & = 7\sqrt{3} \end{aligned} \right. \\
 = 2\sqrt{x} &
 \end{aligned}$$

MCF 3MI

7.5 Exploring the Properties of Exponential Functions

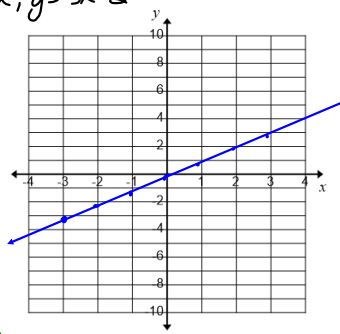
Comparing Linear, Quadratic and Exponential Functions

A) Linear $g(x) = x$

Other examples: $y = 3x$, $y = x^2$, $y = 2^x$

Date: May 16/18

| x | $g(x) = x$ | 1 st diff. | 2 nd diff. |
|----|------------|-----------------------|-----------------------|
| -3 | -3 | $2 - (-3) = 5$ | $5 - 0 = 5$ |
| -2 | -2 | $1 - (-2) = 3$ | |
| -1 | -1 | $0 - (-1) = 1$ | |
| 0 | 0 | $1 - 0 = 1$ | |
| 1 | 1 | $2 - 1 = 1$ | |
| 2 | 2 | $3 - 2 = 1$ | |
| 3 | 3 | $4 - 3 = 1$ | |



\therefore the 1st difference are constant
 \therefore the table represents a linear relation

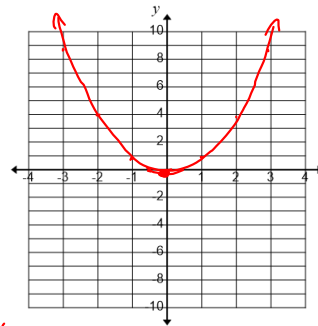
Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$

B) Quadratic $h(x) = x^2$

Other examples: $y = 2x^2$, $y = 3(x-4)^2 + 7$

| x | $h(x) = x^2$ | 1 st diff. | 2 nd diff. |
|----|--------------|-----------------------|-----------------------|
| -3 | $(-3)^2 = 9$ | $4 - 9 = -5$ | $-3 - (-5) = 2$ |
| -2 | 4 | $1 - 4 = -3$ | |
| -1 | 1 | $0 - 1 = -1$ | |
| 0 | 0 | $1 - 0 = 1$ | |
| 1 | 1 | $4 - 1 = 3$ | |
| 2 | 4 | $9 - 4 = 5$ | $5 - 3 = 2$ |
| 3 | 9 | $16 - 9 = 7$ | $7 - 5 = 2$ |



\therefore 2nd difference are constant
 \therefore the table represents a quadratic relation.

Domain: $\{x \in \mathbb{R}\}$

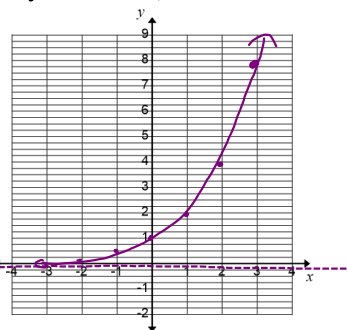
Range: $\{y \in \mathbb{R} \mid y \geq 0\}$

C) Exponential $f(x) = b^x$

Other examples: $y = 3^x$, $y = 2(3^x)$

| x | $f(x) = 2^x$ | 1 st diff. | 2 nd diff. |
|----|------------------------|-------------------------------------------|---------------------------------------------------------------------|
| -3 | $2^{-3} = \frac{1}{8}$ | $\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$ | $\frac{1}{4} \div \frac{1}{8} = \frac{1}{8} \times 4 = \frac{1}{2}$ |
| -2 | $2^{-2} = \frac{1}{4}$ | $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ | |
| -1 | $2^{-1} = \frac{1}{2}$ | $1 - \frac{1}{2} = \frac{1}{2}$ | |
| 0 | $2^0 = 1$ | $2 - 1 = 1$ | |
| 1 | $2^1 = 2$ | $4 - 2 = 2$ | |
| 2 | $2^2 = 4$ | $8 - 4 = 4$ | $4 - 2 = 2$ |
| 3 | $2^3 = 8$ | $16 - 8 = 8$ | $8 - 4 = 4$ |

y-ratio
 $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times 4 = 2$
 $\frac{2}{1} = 2$
 $\frac{4}{2} = 2$ H.A.
 $y = 0$



\therefore y ratios are constant
 \therefore the table is exponential

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y > 0\}$

horizontal asymptote

$$2^{-1} = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

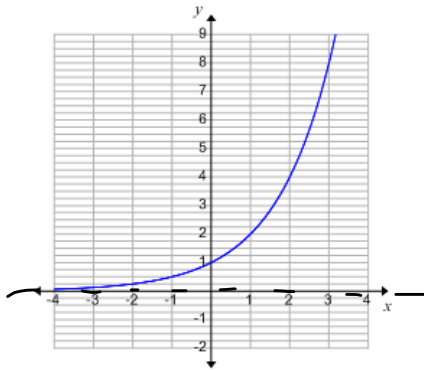
$$2^{-2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

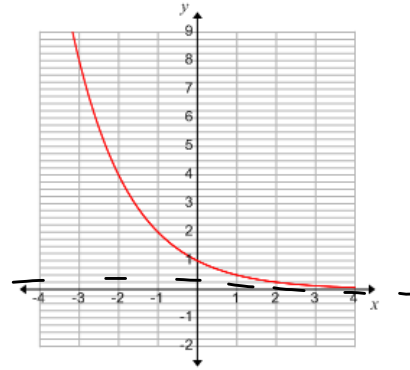
The exponential function $f(x) = b^x$ has the following characteristics:

- it is defined if $b > 0$ and $b \neq 1$ [$b > 0$ means that the base MUST be positive]
- it has a y-intercept of 1
- it has a horizontal *asymptote*, which is the x-axis
- If $b > 1$, the greater the value, the faster the growth
- If $0 < b < 1$, the lesser the value, the faster the decay
- Domain: $\{x \in \mathbb{R}\}$
- Range: $\{y \in \mathbb{R} \mid y > 0\}$

Exponential Growth: $f(x) = b^x, b > 1$



Exponential Decay: $f(x) = b^x, 0 < b < 1$



What are some real-life examples of where these functions exist?

- Compound Interest
- Cell growth
- Bacteria growth
- Population growth
- Radioactive decay

Ex. 1: Use each table of values to identify each function as linear, quadratic or exponential. Justify your answer using difference tables.

a) FD

| x | y |
|----|----|
| -2 | -7 |
| -1 | -4 |
| 0 | -1 |
| 1 | 2 |
| 2 | 5 |

$-4 - (-7) = 3$
 $-1 - (-4) = 3$
 $2 - (-1) = 3$
 $5 - 2 = 3$

∴ First Differences are constant
 ∴ the table represent a linear function.

b) FD

| x | y |
|----|------|
| -2 | -2 |
| -1 | -0.5 |
| 0 | 0 |
| 1 | -0.5 |
| 2 | -2 |

$-5 - (-2) = -1.5$
 $0 - (-0.5) = 0.5$
 $-0.5 - 0 = -0.5$
 $-2 - (-0.5) = -1.5$

SD

$0.5 - 1.5 = -1$
 $-0.5 - 0.5 = -1$
 $-1.5 - (-0.5) = -1$

∴ the 2nd diff are constant
 ∴ the table represents a quadratic.

c) FD y ratios

| x | y |
|----|------|
| -2 | 0.25 |
| -1 | 0.5 |
| 0 | 1.0 |
| 1 | 2.0 |
| 2 | 4.0 |

$0.5 - 0.25 = 0.25$
 $1 - 0.5 = 0.5$
 $2 - 1 = 1$
 $4 - 2 = 2$

$\frac{0.5}{0.25} = 2$
 $\frac{1}{0.5} = 2$
 $\frac{2}{1} = 2$

∴ the y-ratios are constant
 ∴ the table represents an exponential

Today's Homework:

READ p. 422 "In Summary"
 Complete p. 423 # 1, 3

AND