

Are there any Homework Questions you would like to see on the board?

pp. 429-431 # 1-10

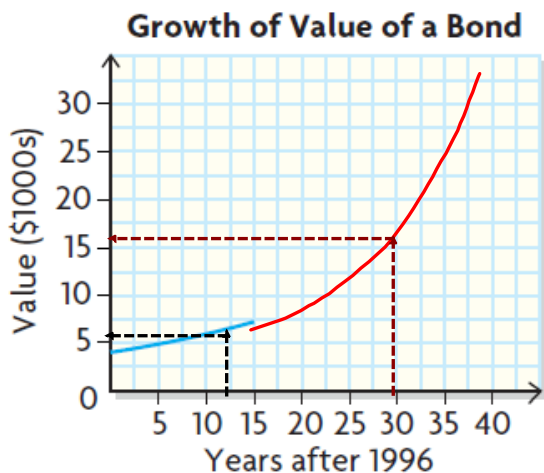
Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) Use exponential functions to model and solve problems involving exponential **decay**.

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1. A brochure for a financial services company has a graph showing the value of a \$4000 savings bond since 1996. Suppose the bond continued to increase in value at the same rate.
- How much will the bond be worth in 2008?
 - How much would it be worth in 2025?
 - Is it possible to determine the length of time needed for the savings bond to double its value from the graph? Explain.



$$\begin{aligned} \text{a) In 2008, } x &= 2008 - 1996 \\ &= 12 \end{aligned}$$

\therefore about \$6000

$$\begin{aligned} \text{a) In 2025, } x &= 2025 - 1996 \\ &= 29 \end{aligned}$$

\therefore about \$16000

c) NO; at least not accurately

MCF 3MI 7.7 Solving Problems Involving Exponential Decay,

$$P(n) = P_0(1-r)^n$$

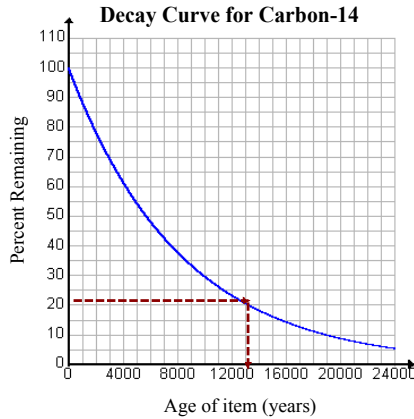
Date: May 23, 2018

Ex. 1: Archaeologists use carbon-14 to estimate the age of the artifacts they discover.

Carbon-14 has a half life of 5730 years.

This means that the amount of carbon-14 in an artifact will be reduced by half every 5730 years.

An ancient animal bone was found at an archaeological dig. Tests were conducted, and the bone was found to contain 20% of the amount of carbon-14 in a present day bone.



Use the decay curve shown to estimate the age of the bone.

From the graph, the bone appears to be about 13 000 years old.

The exponential function $P(n) = P_0(1-r)^n$ can be used as a model to solve problems involving exponential decay, where:

- $P(n)$ is the final amount or number
- P_0 is the initial amount or number
- r is the rate of decay
- n is the number of decay periods

Notes: If the decay rate is given as a percent, it needs to be converted into a fraction or decimal, and then **subtracted** from 1 in the exponential-decay equation. The units for decay rate and decay periods must be compatible (ex. If light intensity decreases per metre, then the number of decay periods must be measured in metres.)

Ex. 2: A tire with a slow puncture loses pressure at the rate of 4%/min. The tire's initial pressure is 300 kPa.

a) Write an equation that models the tire's pressure over time.

$$\begin{aligned} 4\% &= 0.04 & b = 1 - r &= 1 - 0.04 = 0.96 & \therefore P(n) &= 300(1 - 0.04)^n \\ & & & & &= 300(0.96)^n \end{aligned}$$

b) Determine the tire's pressure after: i) 1 min ii) 2 min iii) 10 min
(Round to 2 decimal places)

$$\begin{aligned} \text{i) } P(1) &= 300(0.96)^1 & \text{ii) } P(2) &= 300(0.96)^2 & \text{iii) } P(10) &= 300(0.96)^{10} \\ &= 288 \text{ kPa} & &= 276.48 \text{ kPa} & &= 199.449 \\ & & & & &= 199.45 \text{ kPa} \end{aligned}$$

Ex. 3: The population of Wawa is decreasing 2% per year. The current population of Wawa is 3200.

- a) Write an exponential equation to model the decreasing population.

$$P(t) = 3200(1 - 0.02)^t$$

$$= 3200(0.98)^t$$

- b) Determine the population of Wawa in 15 years.

$$P(15) = 3200(0.98)^{15}$$

$$\approx 2363.4$$

$$\approx 2363 \quad \therefore \text{the population is about } 2363.$$

Ex. 4: Nonsensium-ABC is a radioactive substance with a half-life of 3 days.

Imagine you start with a 100 g sample of Nonsensium-ABC. [Note: This is the same as calculating the percent remaining.]

- a) Write an exponential equation to model the decay of your sample.

$$b = 1 - r$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

$$P(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{3}}$$

- b) Determine the mass of Nonsensium-ABC remaining after 12 days.

$$P(12) = 100\left(\frac{1}{2}\right)^{\frac{12}{3}}$$

$$= 100\left(\frac{1}{2}\right)^4$$

$$= 6.25 \text{ g}$$

$$P = 100(0.5)^4$$

$$100 \times 0.5^4$$

$$(12 \div 3) = 4$$

Ex. 5: Thorium-234 is a radioactive substance with a half-life of 24 days.

If you started with a 280 g sample of Thorium-234, how much would remain after 48 days?

$$P(t) = 280\left(\frac{1}{2}\right)^{\frac{t}{24}}$$

$$= 280\left(\frac{1}{2}\right)^{\frac{48}{24}}$$

$$= 280\left(\frac{1}{2}\right)^2$$

$$= 70 \text{ g}$$

Ex. 6: Radium-226 is a radioactive substance with a half-life of 1600 years.

- a) Write an exponential equation to model this radioactive decay, as a percent.

$$P(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{1600}}$$

- b) What is the concentration of radium-226 remaining on a piece of cloth after 7500 years?
(Round to 3 decimal places.)

$$P(7500) = 100\left(\frac{1}{2}\right)^{\frac{7500}{1600}}$$

$$\approx 3.8808$$

$$\approx 3.881\%$$

Revisit Today's Learning Goals

Homework: pp. 437-439 # 1 - 9

READ pp. 442-443

AND