Date: ____________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) calculate the sum of the terms of an arithmetic series.

Last day's work: pp. 430-432 #1 – 3, 5 – 11 [18-20]

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7.5 Arithmetic Series

Date: May 30, 2018

Recall: An arithmetic sequence is a list of numbers with a common difference.

An **Arithmetic Series** is the **sum** of the terms of an arithmetic sequence.

Sequence: 3, 5, 7, 9, ...
Series: 3 + 5 + 7 + 9 + ...

Karl Friedrich Gauss
(1777-1855)

There's a famous (and probably apocryphal) story about the mathematician Karl Friedrich Gauss that goes something like this:

apocryphal (of a story or statement) of doubtful authenticity, although widely circulated as being true.

Gauss was 9 years old, and sitting in his math class.
He was a genius even at this young age, and as such was incredibly bored in his class and would always goof off and get into trouble.
One day his teacher, J.G. Büttner, wanted to punish him for goofing off, and told him that if he was so smart, why didn't he go sit in the corner and add up all the integers from 1 to 100? Gauss went and sat in the corner, but didn't pick up his pencil.
The teacher confronted him, saying "Karl! Why aren't you working? I suppose you've figured it out already, have you?"
Gauss responded with "Yes – it's 5,050." The teacher didn't believe him and spent the next ten minutes or so adding everything up by hand, only to find that Gauss was right!

So how did Gauss find the answer so fast? What did he see that his teacher didn't?
The answer is simple, really – it's all about pattern recognition.
Let's look at the problem more closely.

Ex.1 Add the following:

$$S_n = 1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100$$

② $$S_n = 100 + 99 + 98 + 97 + \ldots + 4 + 3 + 2 + 1$$ ①

③

④ $$2S_n = 101 + 101 + 101 + \ldots + 101 + 101$$

⑤ $$2S_n = 100\,(101)$$

$$S_n = \frac{100(101)}{2}$$

$$= 5050$$

S_n - partial sum

or the sum of the first n terms

Note: From Gauss, the sum of the first n numbers: $S_n = \dfrac{n(n+1)}{2}$

In general:

$nd-2d+d$
$=nd-d$

$$S_n = a + a+d + a+2d + \dots + a+(n-3)d + a+(n-2)d + \overset{t_n}{a+(n-1)d} \quad ①$$

$$② \; S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d + \dots + a+2d + a+d + a$$

$$2S_n = \underbrace{2a+(n-1)d + 2a+(n-1)d + 2a+(n-1)d + \dots 2a+(n-1)d + 2a+(n-1)d + 2a+(n-1)d}_{n \text{ terms}}$$

$$2S_n = n[2a+(n-1)d]$$

$a+a+(n-1)d$

The Arithmetic Series Formula:

$$S_n = \frac{n}{2}[2a+(n-1)d] \quad \textbf{or} \quad S_n = \frac{n}{2}[t_1 + t_n]$$

where n is the term's position number,
$a = t_1$,
d is the common difference, and
t_n = last term

Ex.2 Find S_{50} for $8 + 11 + 14 + ... + t_n$

a. series

$t_1 = 8$
$= a$

$d = 3$

$n = 50$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{50} = \frac{50}{2}(2(8) + (49)(3))$

$= 25(16 + 147)$

$= 25(163)$

$= 4075$

$S_{50} = 4075$

Ex.3 Find the sum of: $-12 - 7 - 2 + 3 + ... + 138$

$+5 \quad +5$

$n = 31$

$S_{31} = 1953$

a. series

$d = 5$

$a = -12$
(t_1)

$t_n = 138$

$t_n = a + (n-1)d$

$138 = -12 + (n-1)(5)$

$138 = -12 + 5n - 5$

$138 = 5n - 17$

$138 + 17 = 5n$

$155 = 5n$

$n = \frac{155}{5}$

$n = 31$

$S_n = \frac{n}{2}[t_1 + t_n]$

$= \frac{n}{2}[-12 + 138]$

$S_{31} = \frac{31}{2}(-12 + 138)$

$= \frac{31}{2}(126)$

$= (31)(63)$

$= 1953$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 430-432 #1 – 3, 5 – 11 [18-20]
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Today's Homework Practice includes:
pp. 452-453 #(1 – 7)ace, 11, 13 [15,16]

Tomorrow: **SWYK 7.1** (on Sequences)