Before we begin, are there any questions from last day's work?

Today's Learning Goal(s):

By the end of the class, I will be able to:

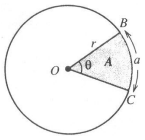
a) calculate the arc length of circle.
b) calculate the area of a sector of a circle.

Cake Decorating assignment due today.

Project Renovation assignment past due.

6.9.1: Arc Length and Sector Area Date: May 29/18

There are problems that often occur in industry that involve arcs and sectors of circles.
Consider the following diagram:



A circle of radius r is drawn, with sector *BOC* bounded by 2 radii, *OB* and *OC*, and an arc *BC*, of length *a*.
The area of the sector is A,
and the sector angle at the centre *O* is θ, measured in degrees.

We can use the proportional relationship: $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{sector area}}{\text{area of circle}} = \frac{\text{sector angle}}{\text{complete rotation}}$

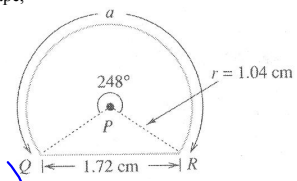
So, if $\frac{a}{2\pi r} = \frac{A}{\pi r^2} = \frac{\theta}{360°}$

then $\frac{a}{2\pi r} = \frac{\theta}{360°}$ and $\frac{A}{\pi r^2} = \frac{\theta}{360°}$ $a = \left(\frac{\theta}{360°}\right) 2\pi r$

and isolating, [*arc length*] $a = 2\pi r\left(\frac{\theta}{360°}\right)$ and [*sector area*] $A = \pi r^2\left(\frac{\theta}{360°}\right)$

Thus both the arc length, *a*, and sector area, *A*, can be calculated
once the radius, *r*, and the sector angle, θ, in degrees, are known.

Ex. 1 A cam for a sewing machine's stitching-control cycle is circular in shape, with a flat side, and has the dimensions shown.
a) Calculate the total perimeter of the cam. (to 3 decimal places)



a) $P = \_\_ + \frown$

$= 1.72 + a$

$\doteq 1.72 + 4.502$

$\doteq 6.222$ cm

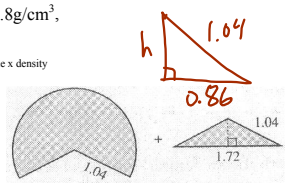
$a = \frac{248}{360} 2\pi r$

$= \frac{248}{360}(2\pi(1.04))$

$\doteq 4.5015$

$\doteq 4.502$

b) If the cam is 0.36 cm thick and is made from an alloy whose density is 3.8g/cm^3, determine the mass of the cam. (to 3 decimal places)
Hint: the cam is a prism, Volume $_{\text{prism}}$ = Area $_{\text{base}}$ x height, $_{\text{then mass = volume x density}}$



$A_{base} = A_{Sector} + A_{Triangle}$

$\doteq 2.341 + 0.503$

$\doteq 2.844$ cm^2

$A_{Sect} = \frac{\theta}{360}(\pi r^2)$

$= \frac{248}{360}\pi(1.04)^2$

$\doteq 2.3408$

$\doteq 2.341$

$A_\Delta = \frac{1}{2}bh$

$\doteq \frac{1}{2}(1.72)(0.586)$

$\doteq 0.5031$

$\doteq 503$

$h^2 = 1.04^2 - 0.86^2$

$h \doteq \sqrt{0.342}$

$\doteq 0.5848$

$\doteq 0.585$

$V = A_{base} \times h$

$\doteq 2.844 \times 0.36$

$\doteq 1.0238$

$\doteq 1.024$ cm^3

$Mass_{Cam} = DV$

$\doteq 3.8 \times 1.024$

$\doteq 3.8912$

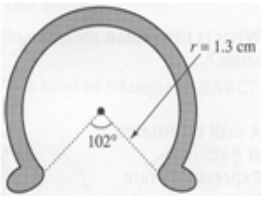
$= 3.891$ g

∴ the mass of the cam is 3.891 g.

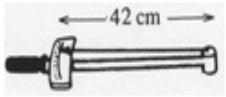
6.9.2 Arc Length and Sector Area

Date: _____

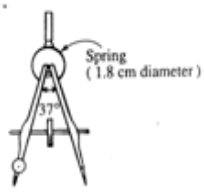
1. A snap-ring retainer clip with dimensions shown is part of a universal joint assembly, and fits snugly around the bearing cap when assembled. What length of the clip is in contact with the bearing cap? (to 2 decimal places)



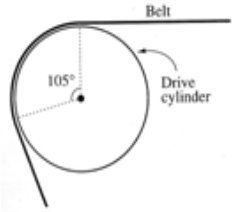
2. The pointer on a torque wrench is 42 cm long, and moves through an angle of 16°. Through what distance does the tip of the pointer move? (to 1 decimal place)



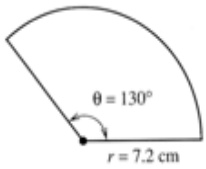
3. Calculate the length of the spring on the bow compasses shown. (to 2 decimal places)



4. The belt on a copier machine is in contact with a drive cylinder over 105° of its surface, as shown. If the length of contact is 5.62 cm, what is the radius of the drive cylinder? (to 2 decimal places)



5. The vent cover on a forced air heating system is in the form of a sector of a circle. Determine the area of sheet metal used to make the vent cover. (to 1 decimal place)



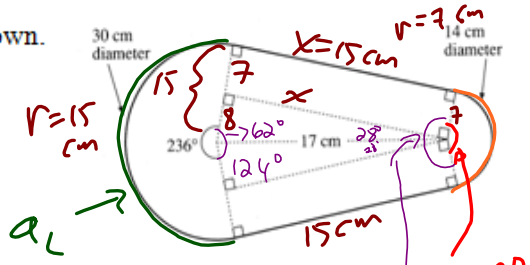
$$A_{sector} = \frac{\theta}{360^\circ}\pi r^2$$

$$= \left(\frac{130^\circ}{360^\circ}\right)\pi(7.2)^2$$

$$\doteq 58.81$$

$$\doteq 58.8 \text{ cm}^2$$

6. The safety shield for a motor pulley drive has the dimensions shown.
   a) Calculate the perimeter of the shield. (to 1 decimal place)
   b) Determine the area of the shield. (to 1 decimal place)



a) $a_L = \left(\frac{236^\circ}{360^\circ}\right) 2\pi(15)$

$\doteq 61.7846$

$\doteq 61.785$ cm

$a_S = \left(\frac{124}{360}\right) 2\pi(7)$

$\doteq 15.1494$

$\doteq 15.149$

$x^2 = 17^2 - 8^2$

$x = 15$ cm

124°?

$180^\circ + 56^\circ = 236^\circ$

∴ Perimeter

$\doteq 15 + 61.785 + 15 + 15.149$

$\doteq 106.934$ cm

Answers

1. 5.85 cm
2. 11.7 cm
3. 5.07 cm
4. 3.07 cm
5. 58.8 cm^2
6. (*a*) 106.9 cm
   (*b*) 846.4 cm^2

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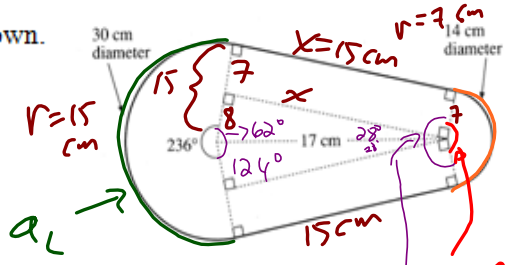

b) $A_{total} = A_{sector_L} + A_{sector_S} + 2A_{TRAP.}$

$= \left(\frac{236}{360}\right)\pi(15^2) + \left(\frac{124}{360}\right)\pi(7)^2 + 2\left(\frac{1}{2}\right)(h)(a+b)$

$\doteq 463.38 + 53.02 + 330$

$\doteq 846.408$

$\doteq 846.4\ cm^2$

$x^2 = 17^2 - 8^2$

$x = 15$ cm

$180° + 56° = 236°$

124°?

Answers

1. 5.85 cm
2. 11.7 cm
3. 5.07 cm
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