Date: ____________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) contrast simple and compound interest.
b) calculate simple interest.
c) calculate the "future value" of a principal with compound interest.

Last day's work: pp. 468-469 #3, 4, 7 – 10, (14 – 16)ace, 18ace, 19b, 20, 22, 23ace
(Review) p. 470 #1 – 5, 7, 8

9b, 14e

p. 468

9. i) Determine the type of each sequence (arithmetic, geometric, or neither), where $n \in \mathbf{N}$.

ii) State the first five terms.

b) $t_n = \dfrac{1}{7n - 3}$

$t_1 = \frac{1}{7(1)-3} = \frac{1}{4}$ $t_2 = \frac{1}{7(2)-3} = \frac{1}{14-3} = \frac{1}{11}$

$t_3 = \frac{1}{7(3)-3} = \frac{1}{21-3} = \frac{1}{18}$

$t_4 = \frac{1}{25}$; $t_5 = \frac{1}{32}$

14. For each arithmetic series, calculate the sum of the first 50 terms.

e) 17.5 + 18.9 + 20.3 + ...

$a = 17.5$

$d = 18.9 - 17.5 = 1.4$

$n = 50$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{50}{2}[2(17.5) + 49(1.4)]$

$= 25[35 + 68.6]$

$= 2590$

2, 6, 18 $\quad \frac{6}{2} = 3 \quad \frac{18}{6} = 3$

3 5 7 $\quad 5-3 = 2 \quad 7-5 = 2$

$\frac{1}{4}, \frac{1}{11}, \frac{1}{18}$

$\frac{1}{11} \div \frac{1}{4} = \frac{1}{11} \times \frac{4}{1} = \frac{4}{11}$ $\quad \frac{1}{18} \div \frac{1}{11} = \frac{1}{18} \times \frac{11}{1} = \frac{11}{18}$

$\frac{1}{11} - \frac{1}{4} = \frac{4-11}{44} = \frac{-3}{44}$ $\quad \frac{1}{18} - \frac{1}{11} = \frac{11-18}{198} = \frac{-7}{198}$

f) $\left(3f^2 - \frac{2}{f}\right)^4$ $\quad (-2f^{-1})$

1 3 3 1
1 4 6 4 1

$= 1(3f^2)^4\left(-\frac{2}{f}\right)^0 + 4(3f^2)^3\left(-\frac{2}{f}\right)^1 + 6(3f^2)^2\left(-\frac{2}{f}\right)^2 + 4(3f^2)^1\left(-\frac{2}{f}\right)^3 + 1(3f^2)^0\left(-\frac{2}{f}\right)^4$

$= (81f^8) + 4(27f^6)\left(\frac{-2}{f}\right) + 6(9f^4)\left(\frac{4}{f^2}\right) + 4(3f^2)\left(\frac{-8}{f^3}\right) + \left(\frac{16}{f^4}\right)$

$= 81f^8 - 216f^5 + 216f^2 - \frac{96}{f} + \frac{16}{f^4}$

8.1 Simple Interest
8.2 Compound Interest (Future Value)

Date: June 5/18

Ex.1 Amanda invests \$500 at 8% /a simple interest. ← per annum

a) Calculate the interest earned after 5 years.

↳ $I = Prt$ → time → years
↳ rate → decimal

$I = ?$
$P = 500$
$r = 0.08$
$t = 5$

$I = Prt$
$= 500(0.08)(5)$
$= 200$

I = \$200

b) Determine the total amount of her investment after 5 years.

$A = P + I$
$= 500 + 200$
$= \$700$

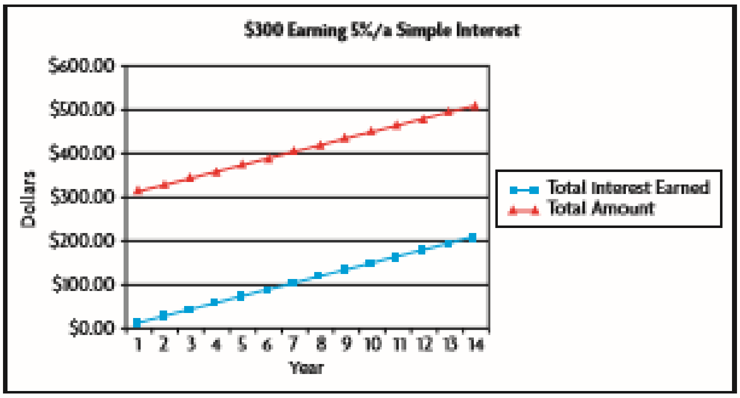
$A = P + I$
$= P + Prt$
$= P(1 + rt)$

A = \$700

Note: Simple interest represents **linear** growth.
The function that models Amanda's investment is:

$f(x) = 40x + 500$ or in general: $f(x) = (Pr)x + P$

Simple interest is calculated only on the principal.
The total amount, *A*, and interest earned, *I*, are linear functions in terms of time, so their graphs are straight lines (see graph below). The values of *A* and *I* at the end of each interest period form the terms of two arithmetic sequences.



Ex.2 Amy invests $500 at 8% /a compounded annually.

a) Determine the total amount of her investment after 5 years.

$A = P(1 + rt)$

0 1 2 3 4 5

500 $A = 500(1 + 0.08(1))$
$= 500(1.08)$
$= 540$

$A = 540(1.08)$
$\doteq \$583.20$

$A = 500(1.08)(1.08)$
$= 500(1.08)^2$

$A = 500(1.08)^5$
$\doteq 734.664$
$\doteq \$734.66$

A = $734.66

Ex.3 Ariel invests $500 at 8% /a compounded quarterly.

a) Determine the total amount of her investment after 5 years.

$A = P + I$

$I = A - P$

0 1 2 3 4 5

0 1 2 3 4 5 6 7 8 ... 18 19 20

$A = P(1+i)^n$

500

$A = P(1 + rt)$
$= 500(1 + 0.08(\frac{1}{4}))$
$= 500(1 + 0.02)$
$= 500(1.02)$
$\doteq \$510$

Growth Factor (per compounding period)

$A = 500(1.02)^2$

$A = 500(1.02)^{20}$
$\doteq 742.973$
$\doteq \$742.973$

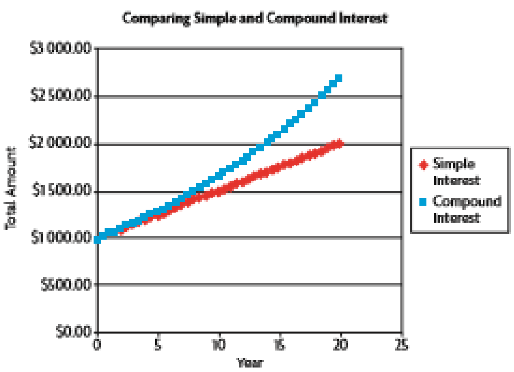
A = $742.97

Note: Compound interest represents **exponential** growth.
The function that models Amy's investment is:

$f(x) = 500(1.08)^x$ or in general: $f(x) = ab^x$

For Ariel: $f(x) = 500(1.02)^{4x}$

Compound interest is calculated by applying the interest rate to the principal and any interest already earned.
The total amounts at the end of each interest period form a geometric sequence. So compound interest results in exponential growth.
The total amount, A, of an investment after a certain period is called the future value of the investment.



Simple and Compound Interest Formulae

- Simple Interest
  $I = Prt$
  $A = P(1 + rt)$
- Compound Interest
  $A = P(1 + i)^n$
  i = interest rate per compounding period
  n = number of compounding periods

P=Principal ($ at start)
A=Amount ($ at the end)

Ex.4

a) Determine the future value of $1800 invested at 6% /*a* compounded semi-annually for 20 years.

$A = ?$
$P = 1800$
$i = \frac{0.06}{2} = 0.03$
$n = 20 \times 2 = 40$

$A = P(1+i)^n$
$= 1800\left(1 + \frac{0.06}{2}\right)^{40}$
$\doteq 5871.668$
$\doteq \$5871.67$

A = $5871.67

b) How long will it take for this investment to at least double?

$A = 3600$
$P = 1800$
$i = \frac{0.06}{2} = 0.03$
$n = x \times 2 = 2x$

$A = P(1+i)^n$
$3600 = 1800(1.03)^{2x}$
$\frac{3600}{1800} = (1.03)^{2x}$
$2 = 1.03^{2x}$

logs?

$\log 2 = \log 1.03^{2x}$
$\log 2 = 2x \log 1.03$
$\frac{\log 2}{\log 1.03} = 2x$
$\frac{\log 2}{2\log 1.03} = x$
$x \doteq 11.724$

$\therefore$ it takes about 11.72 years to double

Trial + Error
$1.03^8 \doteq 1.266$
$1.03^{20} \doteq 1.806$
$1.03^{23} \doteq 1.973$
$1.03^{23.4} \doteq 1.997$
$\therefore 2x \doteq 23.4$
$x \doteq 11.7$ years.

about 11.72 years

For present value questions, ← Thursday's Lesson

$A = 587.67$ you are looking to see how much to invest to get the desired amount at the end.

$P = ?$

$i = \frac{0.06}{2}$

$n = 20 \times 2$
$= 40$

* Reversing Ex. 4a)

$$A = P(1+i)^n$$

$$587.67 = P\left(1 + \frac{0.06}{2}\right)^{40}$$

$$\frac{587.67}{(1.03)^{40}} = P$$

$$P \doteq 1800.0006$$
$$\doteq \$1800.00$$ (which we expected)

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 468-469 #3, 4, 7 – 10, (14 – 16)ace,
18ace, 19b, 20, 22, 23ace
p. 470 #1 – 5, 7, 8

Read the Key Ideas/Need to Know
pp.480-481 and p.489

Study for the Unit 7 Summative!!

Today's Homework Practice includes:
pp. 481-482 #5 – 10
pp. 490-492 #4 – 9, 11, 14 [20]