

Date: _____

Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) calculate the "present value" of an annuity earning compound interest.

Quiz tomorrow

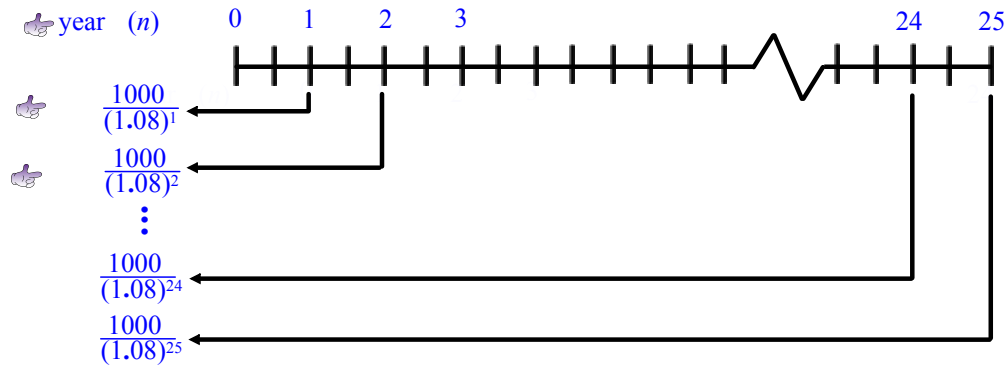
Last day's work: pp. 511-512 #2, 5ac, 6, 7

8.5 Annuities: Present Value

Date: June 11/18

Ex.1 In June, Ms. Sterlo decides to deposit some money into a fund to provide a \$1000 commencement scholarship at the end of each school year for the next 25 years. The fund will pay 8% /a compounded annually. How much needs to be invested now?

$$A = P(1+i)^n \quad \therefore P = \frac{A}{(1+i)^n}$$



(It is easier to start at the bottom; it makes "r" nicer.)

$$S_{25} = \frac{1000}{(1.08)^{25}} + \frac{1000}{(1.08)^{24}} + \dots + \frac{1000}{(1.08)^2} + \frac{1000}{(1.08)}$$

This is a **Geometric Series**, with $a = \frac{1000}{(1.08)^{25}}$, $r = 1.08$, $n = 25$

Use $S_n = \frac{a(r^n - 1)}{r - 1}$

$$\therefore S_{25} = \frac{\frac{1000}{(1.08)^{25}} (1.08^{25} - 1)}{(1.08 - 1)}$$

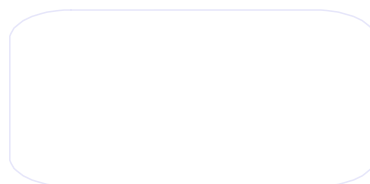
$$\approx 10\,674.776$$

$$= \$10\,674.78$$

\therefore a sum of \$10 674.78 invested now would provide a payment of \$1000 for each of the next 25 years.

Discuss Interest earned?

$$\begin{array}{r} \$1000 \times 25 \\ = \$25\,000 \\ - \$10\,674.78 \\ \hline \$14\,325.22 \end{array}$$

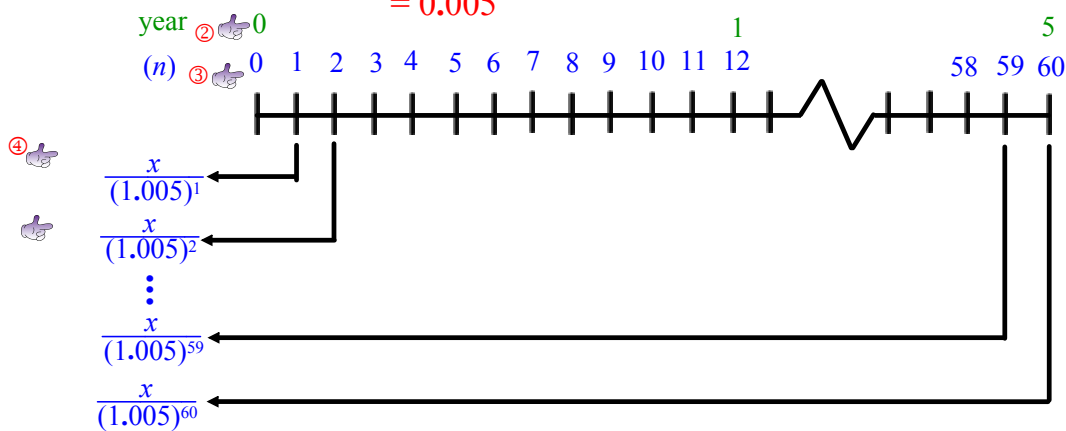


Ex.2

At the end of university you owe \$20 000.

Determine your monthly payment at 6% /a compounded monthly for 5 years.

$$\text{Use } P = \frac{A}{(1+i)^n} \quad \textcircled{1} \text{ } i = \frac{0.06}{12} = 0.005$$



$$\textcircled{5} \text{ Use } S_n = \frac{a(r^n - 1)}{r - 1} \quad \textcircled{6} \text{ } a = \frac{x}{(1.005)^{60}}, r = 1.005, n = 60, S_{60} = 20\,000$$

$$\textcircled{7} \text{ } 20\,000 = \frac{\frac{x}{(1.005)^{60}} (1.005^{60} - 1)}{1.005 - 1} \quad \leftarrow .005$$

Solve for x .

$$\textcircled{8} \text{ } 20\,000 (.005) = \frac{x}{(1.005)^{60}} (1.005^{60} - 1)$$

$$\textcircled{9} \text{ } 20\,000 (.005) (1.005)^{60} = x (1.005^{60} - 1)$$

$$\textcircled{10} \text{ } \frac{20\,000 (.005) (1.005)^{60}}{(1.005^{60} - 1)} = x$$

$$\textcircled{11} \text{ } x \doteq 386.656$$

$$\textcircled{12} \text{ } = \$386.66$$

\therefore the monthly payment is \$386.66

If time, discuss Interest paid, and my "offer"?

$$\begin{aligned} & \$386.66 \times 60 \\ & = \$23\,199.60 \end{aligned}$$

Another formula:

$$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$$

where R is the regular payment
i is the interest rate per compound period
n is the number of compound periods

Read p. 518 Example 3

Read the Key Ideas/Need to Know p.519

Today's Homework Practice includes:

pp. 520-521 #1, 2b, 3ac, 5, 7

Quiz tomorrow