1.3 Properties of Graphs of Functions

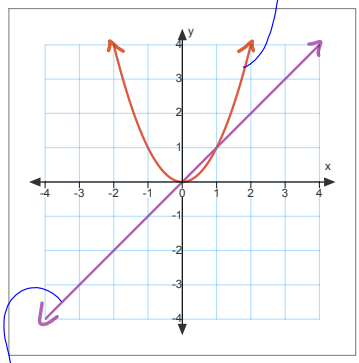
Math Learning Target:
"I can compare properties between parent functions, and within a parent function's family."

A **transformation** is a geometric operation, such as a translation, reflection and compression.

Each transformation is performed on a parent relation. There are many parent relations.
A **parent function** belongs to the set of parent relations and is the simplest function in a family of functions.
For example, the family of quadratic functions are all constructed from $y = x^2$.

Here are the *seven* parent functions that will be used often:

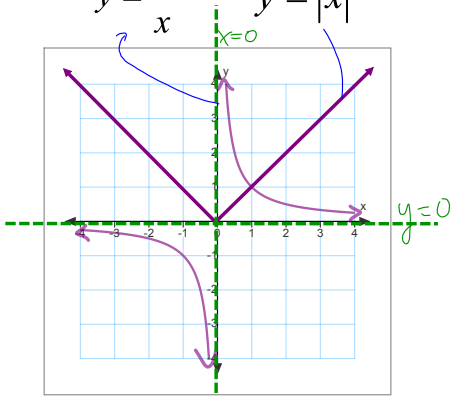
$y = x$ $\quad$ $y = x^2$



$y = x$
Interval(s) of increase: $(-\infty, \infty)$
Interval(s) of decrease: NONE
End behaviours:
as $x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

$y = x^2$
Interval(s) of increase: $[0, \infty)$
Interval(s) of decrease: $(-\infty, 0)$
End behaviours:
as $x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

$y = \frac{1}{x}$ $\quad$ $y = |x|$

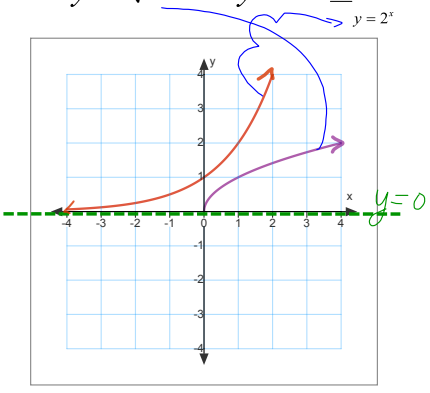


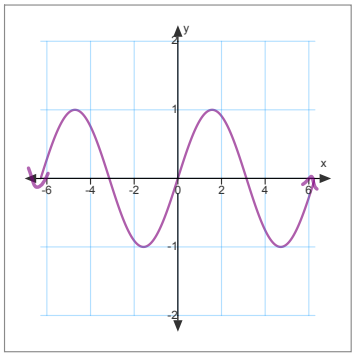
$y = \frac{1}{x}$
Interval(s) of increase: NONE
Interval(s) of decrease: $(-\infty, 0) \cup (0, \infty)$
End behaviours: "OR"
as $x \to \infty, y \to 0$
$x \to -\infty, y \to 0$

$y = |x|$
Interval(s) of increase: $[0, \infty)$
Interval(s) of decrease: $(-\infty, 0)$
End behaviours:
as $x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

***Recall: Did you include "0" in one interval OR the other.**

$y = \sqrt{x}$ $\quad$ $y = b^x$ i.e. $b = 2$ $\quad$ $y = \sin(x)$





$y = \sqrt{x}$
Interval(s) of increase: $[0, \infty)$
Interval(s) of decrease: NONE
End behaviours:
as $x \to \infty, y \to \infty$

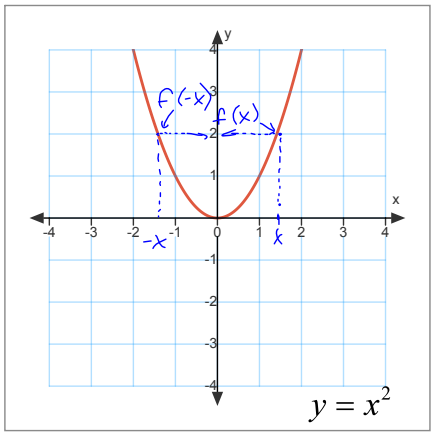
$y = \sin(x)$ End behaviours:
as $x \to \pm\infty$ y oscillates between $y = 1$ and $y = -1$.

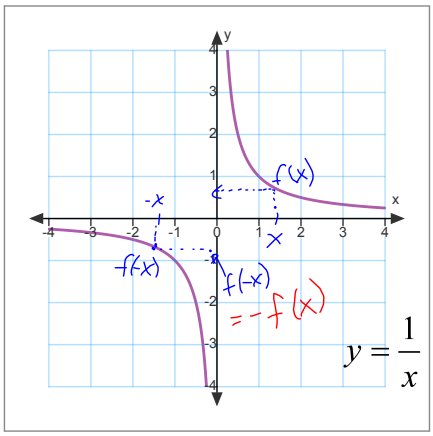
$y = 2^x$
Interval(s) of increase: $(-\infty, \infty)$
Interval(s) of decrease: NONE
End behaviours:
as $x \to \infty, y \to \infty$
$x \to -\infty, y \to 0$

CHALLENGE! Can you determine expressions for the intervals of increase and intervals of decrease?

"Symmetry"

A function is **odd** when $f(-x) = -f(x)$ A function is **even** when $f(-x) = f(x)$





Graphically, a function is even when it is symmetric about the y-axis (only).

Graphically, a function is odd when it is symmetric about both axes
it has symmetry about the origin
<rotational symmetry of 180°
double reflection → reflect in x, then in y

$f(x) = x^2$

Ex.1 Is $y = x^2$ even, odd, or neither? Prove algebraically.

Consider $f(-x)$
$= (-x)^2$
$= (-x)(-x)$
$= x^2$
$= f(x)$

$\therefore f(-x) = f(x)$
$\therefore f(x)$ is EVEN

Ex.2 Is $y = \frac{1}{x}$ even, odd, or neither? Prove algebraically.

Consider $f(-x)$
$= \frac{1}{(-x)}$
$= -\frac{1}{x}$
$= -f(x)$

$\therefore f(-x) = -f(x)$
$\therefore f(x)$ is ODD.

Do: pg. 23 #3*, 4ad, 5**, 6, 7, 8, 10***, 15

* Error in answer: the function can be derived from any $y=b^x$, for any valid "b"),

** The instructions are poor. Simply apply what was learned today in the lesson.

***In #10a, in the instructions for the question change $(-\infty, -2)$ to $(-\infty, 2]$

positive 2

square bracket

YES, you have permission to write in the textbook to make this change!

P. 11-13

2d) $y = \cos t + 1$