1.5 Inverse Relations

Math Learning Target:
"I know how to find the equation and graph of an inverse relation,
and I can state its properties.
Also, I know under what conditions the inverse relation is a function."

Simply stated, an **inverse** is something that is the opposite or reverse of something else. For example, the inverse of the operation addition is subtraction and vice versa. When a mathematical operation does something with terms, its inverse operation undoes it.

Do: $5 \to \boxed{\text{mult by 4}} \to 20$

Undo: $20 \to \boxed{\text{divide by 4}} \to 5$

The idea of an inverse applies to relations too! If the relation is a function, a function accepts one input and produces one output.
The **inverse function** accepts that output (as an input) and produces one output (the original function's input!).

Do: $x \to \boxed{\text{mult. by 4}} \to 4x$

Undo: $4x \to \boxed{\text{divide by 4}} \to x$

Note: not all inverse relations are functions.

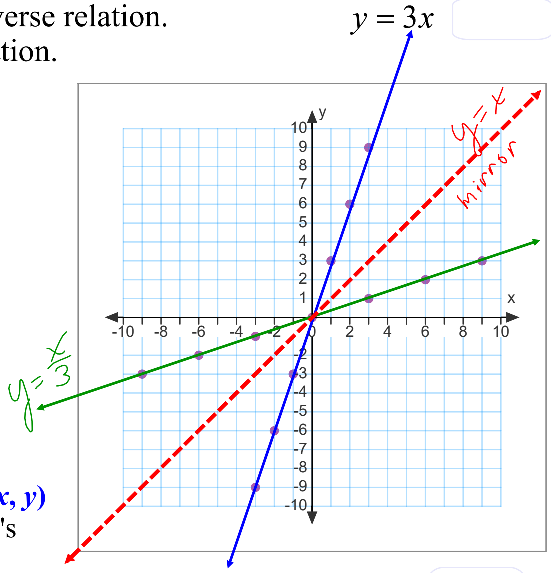
Ex. 1:
a) Using a table of values, graph $y = 3x$
b) Using a table of values, graph its inverse relation.
c) State the equation of the inverse relation.

a)

x (old)	$y = 3x$ (old)
-3	-9
-2	-6
-1	-3
0	0
1	3
2	6
3	9

b)

x_{NEW} (y_{OLD})	y_{NEW} (x_{OLD})
-9	-3
-6	-2
-3	-1
0	0
3	1
6	2
9	3



Notice: The "old" function's ordered pairs (x, y) map to the "new" (inverse) function's ordered pairs (y, x).

c) $y = 3x$

$x \rightleftarrows y$

$x = 3y$

$\frac{x}{3} = y$

$f^{-1}(x) = \frac{x}{3}$ or $f^{-1}(x) = \frac{1}{3}x$

Note: An inverse relation is a reflection of the original relation in $y = x$.

Ex. 2:

a) Determine the equation of the inverse relation of $f(x) = (x-3)^2 + 4$

b) Without graphing, is the inverse relation a function? Explain.

a) $y = (x-3)^2 + 4$

$x \rightleftarrows y$

$x = (y-3)^2 + 4$

$x - 4 = (y-3)^2$

$\pm\sqrt{x-4} = y - 3$

$\pm\sqrt{x-4} + 3 = y$

$\cancel{f^{-1}(x) = \pm\sqrt{x-4} + 3}$

NO!!

b) For (*almost*) every element in the Domain, there are 2 corresponding elements in the Range, therefore, the inverse relation is NOT a function.

Ex. 3: Given: $h(x) = 2x^3$
Find: $h^{-1}(-8)$

$x = 2y^3$

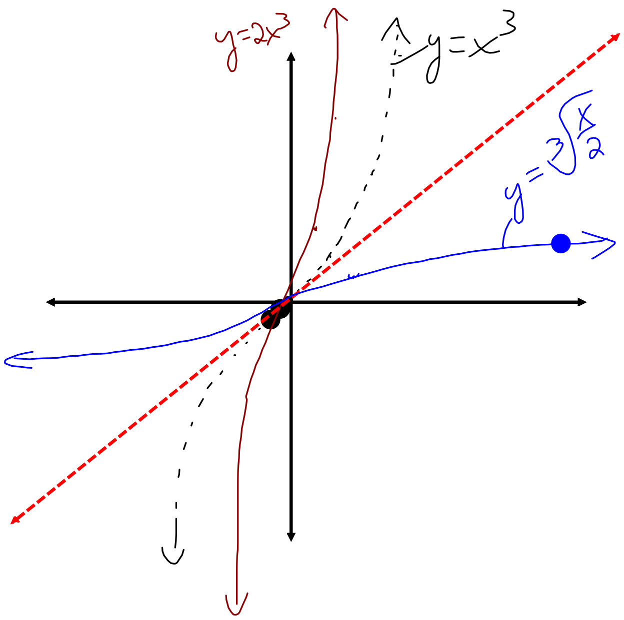
$\frac{x}{2} = y^3$

$\sqrt[3]{\frac{x}{2}} = y$

$h^{-1}(x) = \sqrt[3]{\frac{x}{2}}$

$h^{-1}(-8) = \sqrt[3]{\frac{(-8)}{2}}$

$= \sqrt[3]{-4}$



All properties of the independent variable in a relation correspond to the properties of the dependent variable in its inverse, and vice versa.

Entertainment: pp. 43-45 #1cd, 2d, 3, 4**, 6d, 10e, 12c, 13ab, 14, 16

To start #4, create a table of values for $y = x^3$ then graph it.

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Optional Quizzes

http://courseware.cemc.uwaterloo.ca/8/assignments/113/4

http://courseware.cemc.uwaterloo.ca/8/assignments/113/5

Quiz Answers Incorrect?

http://courseware.cemc.uwaterloo.ca/8/assignments/113/6

http://courseware.cemc.uwaterloo.ca/8/assignments/113/7

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9. (2, 1) is a point on the graph of $y = f(x)$. Find the corresponding point on the graph of each of the following functions.

a) $y = -f(-x) + 2$
b) $y = f(-2(x + 9)) - 7$
c) $y = f(x - 2) + 2$
d) $y = 0.3f(5(x - 3))$
e) $y = 1 - f(1 - x)$
f) $y = -f(2(x - 8))$

d) $y = 0.3f(5(x-3))$

$(x,y) \to \left(\frac{1}{5}x+3,\ 0.3y\right)$

$(2,1) \to \left(\frac{1}{5}(2)+3,\ 0.3(1)\right)$

$\to (3.4,\ 0.3)$

$\to \left(\frac{2}{5}+\frac{15}{5},\ \frac{3}{10}\right)$

$\to \left(\frac{17}{5},\ \frac{3}{10}\right)$

e) $y = -f(1-x) + 1$

$= -f(-x+1) + 1$

$= -f(-(x-1)) + 1$

$(x,y) \to (-x+1,\ -y+1)$

$(2,1) \to (-(2)+1,\ -(1)+1)$

$\to (-1,\ 0)$