

Correct Yesterday's entertainment:
pp. 51-53 #1bdf, 2bdf, 3a, 4a, 5d, 7, 9, 14, 15

Unit 1 Review Ideas **WITH proper Mathematical notation/form**

describing transformations

knowing your parent functions!!

domain and range; interval notation or set notation
end behaviours AND intervals of increase or decrease

absolute value notation $\{x \in \mathbb{R} \mid |x| < 7\}$

mapping a point

testing for even vs. odd vs. neither algebraically, starting with $f(-x)$

inverses of functions

piecewise functions



inc $(-\infty, 1] \cup [3, \infty)$ dec.
 $(1, 3]$

- p. 52 5. Graph the following piecewise functions. Determine whether each function is continuous or not, and state the domain and range of the function.

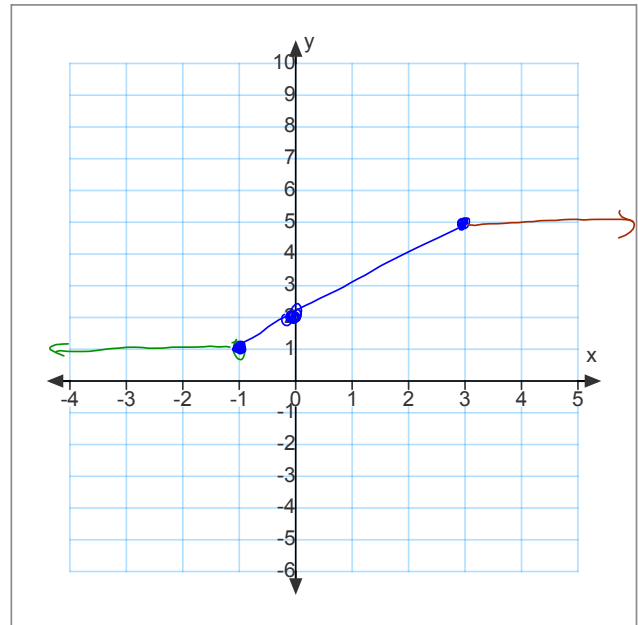
$$d) f(x) = \begin{cases} 1, & \text{if } x < -1 \\ x + 2, & \text{if } -1 \leq x \leq 3 \\ 5, & \text{if } x > 3 \end{cases}$$

$$\begin{aligned} f(-1) &= x + 2 \\ &= (-1) + 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} f(3) &= 3 + 2 \\ &= 5 \end{aligned}$$

$$\begin{aligned} D: & (-\infty, \infty) \\ R: & [1, 5] \end{aligned}$$

$f(x)$ is
continuous



p. 52

7. Many income tax systems are calculated using a tiered method. Under a certain tax law, the first \$100 000 of earnings are subject to a 35% tax; earnings greater than \$100 000 and up to \$500 000 are subject to a 45% tax. Any earnings greater than \$500 000 are taxed at 55%.

Write a piecewise function that models this situation.

$$f(x) = \begin{cases} 0.35x & 0 \leq x \leq 100\,000 \\ 0.45x - 10\,000 & 100\,000 < x \leq 500\,000 \\ 0.55x - 60\,000 & x > 500\,000 \end{cases}$$

if $x = 100\,000$

$$f(x) = 0.35x$$

$$= 0.35(100\,000)$$

$$= 35\,000$$

$$f(x) = 35\,000 + 0.45(x - 100\,000)$$

$$= 35\,000 + 0.45x - 45\,000$$

$$= 0.45x - 10\,000$$

$$f(x) = 215\,000 + 0.55(x - 500\,000)$$

$$= 215\,000 + 0.55x - 275\,000$$

$$= 0.55x - 60\,000$$

if $x = 500\,000$

$$f(500\,000) =$$

$$= 0.45(500\,000) - 10\,000$$

$$= 215\,000$$

p. 52

9. The fish population, in thousands, in a lake at any time, x , in years is modelled by the following function:

$$f(x) = \begin{cases} 2^x, & \text{if } 0 \leq x \leq 6 \\ 4x + 8, & \text{if } x > 6 \end{cases}$$

This function describes a sudden change in the population at time $x = 6$, due to a chemical spill.

- Sketch the graph of the piecewise function.
- Describe the continuity of the function.
- How many fish were killed by the chemical spill?
- At what time did the population recover to the level it was before the chemical spill?
- Describe other events relating to fish populations in a lake that might result in piecewise functions.

if $x=6$ $f(6) = 2^6 = 64$
 if $x=6$ $f(6) = 4(6) + 8 = 24 + 8 = 32$
 difference = $64 - 32 = 32$
 (thousand)

b) discontinuous
 \therefore there is a jump at $x=6$

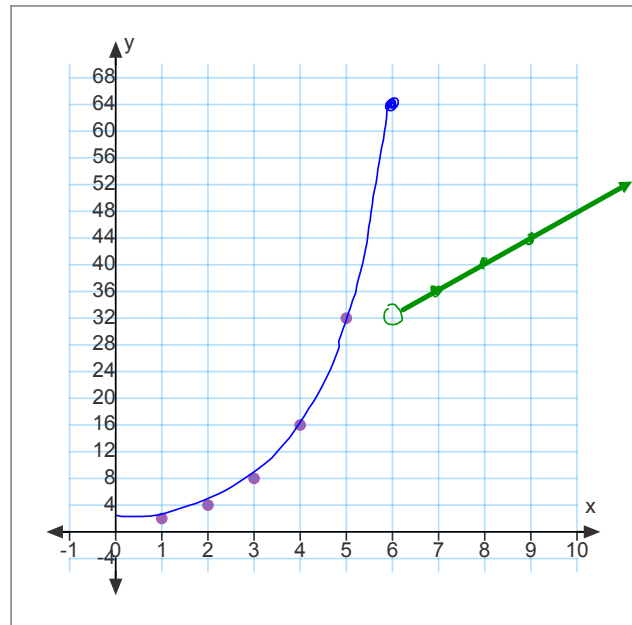
c) 32 000 fish were killed by the chemical spill.

d) when does $f(x) = 4x + 8$
 (the linear function) equal 64.

$$64 = 4x + 8$$

$$56 = 4x$$

$$x = 14$$



p. 53 14. Explain why there is no value of k that will make the following function continuous.

$$f(x) = \begin{cases} 5x, & \text{if } x < -1 \\ x + k, & \text{if } -1 \leq x \leq 3 \\ 2x^2, & \text{if } x > 3 \end{cases}$$

at $x = -1$

$$\begin{aligned} f(x) &= 5x \\ f(-1) &= 5(-1) \\ &= -5 \end{aligned} \quad \left. \begin{aligned} f(x) &= x + k \\ f(-1) &= (-1) + k \\ &= k - 1 \end{aligned} \right\}$$

$$-5 = k - 1$$

$$-5 + 1 = k$$

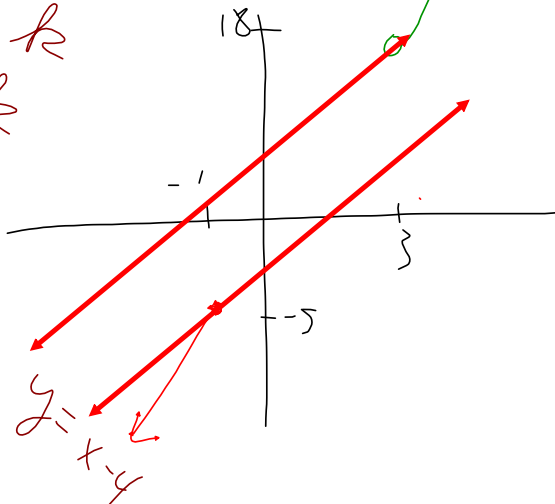
$$-4 = k$$

at $x = 3$

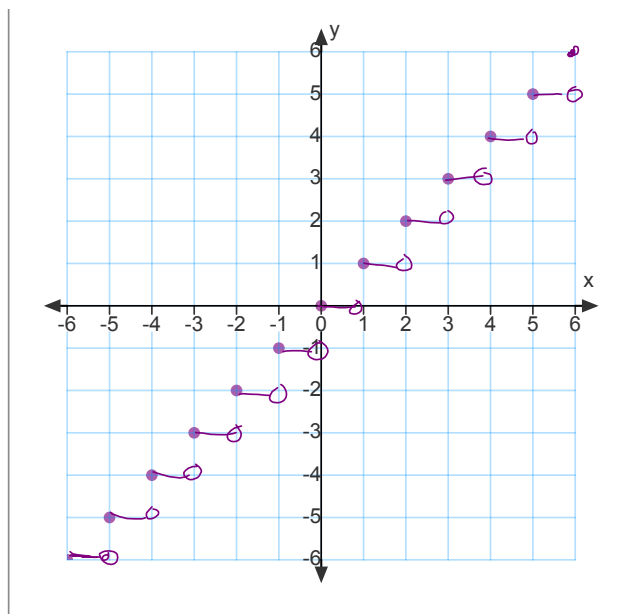
$$\begin{aligned} f(x) &= x + k \\ f(3) &= 3 + k \\ \text{if } k &= -4 \\ \therefore f(3) &= 3 + (-4) \\ &= -1 \end{aligned} \quad \left. \begin{aligned} f(x) &= 2x^2 \\ &= 2(3)^2 \\ &= 2(9) \\ &= 18 \end{aligned} \right\}$$

$$-1 \neq 18$$

discontinuous at $x = 3$



- p. 53 15. The *greatest integer function* is a step function that is written as $f(x) = [x]$, where $f(x)$ is the greatest integer less than or equal to x . In other words, the greatest integer function rounds any number down to the nearest integer. For example, the greatest integer less than or equal to the number $[5.3]$ is 5, while the greatest integer less than or equal to the number $[-5.3]$ is -6 . Sketch the graph of $f(x) = [x]$.



15.