Before we begin, are there any questions from last day's work?

p. 344 # 7, 10(a,b), 11

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) Convert between the exponential and logarithmic forms of an equation
b) Solve an exponential equation by **"taking the log of both sides"**.

NOTE: There is no handout for today's lesson.
If you did not print the pages in advance as advised,
then copy these examples onto lined paper in your notebook.

Express 8 as a power of 2.

$2^x = 8$

$x = 3$ • $2^3 = 8$

p. 344 **10. Express:**

a) 7 as a power of 3

$3^x = 7$

$\log_3 7 = x$

$x = \frac{\log 7}{\log 3}$

$\doteq 1.7712$

$\doteq 1.771$

b) 5 as a power of 2

$2^y = 5$

$\log_2 5 = y$

$y = \frac{\log 5}{\log 2}$

$\doteq 2.3219$

$\doteq 2.322$

1.8.0 Warm-up

Date: Sept. 17/18

1. Complete the chart:

Exponential Equation	Logarithmic Equation
$2^5 = 32$	$\log_2 32 = 5$
$3^4 = 81$	$\log_3 81 = 4$
$10^3 = 1000$	$\log_{10} 1000 = 3$
$2^x = 256$	$\log_2 256 = x$
$2^6 = 64$	$\log_2 64 = 6$
$2^y = 8$	$\log_2 8 = y$

1.8.0 Warm-up (cont'd)

2. Solve for x. (Round to 3 decimal places)

a) $2^x = 18$

$\log_2 18 = x$

$x = \frac{\log 18}{\log 2}$

$\doteq 4.1699$

$\doteq 4.170$

b) $3^x = 25$

$\log_3 25 = x$

$x = \frac{\log 25}{\log 3}$

$\doteq 2.9299$

$\doteq 2.930$

Check

$2^{4.170}$

$\doteq 18.00$

4.1699

2.9299

1.8.1: Solving Exponential Equations Using Logarithms

Date: Sept. 17/18

Laws of Logarithms for Powers $\log_a x^n = n\log_a x \quad [x>0, a>0, a\neq 1]$

Ex. 1 $\log 8 = 0.903$ and

$$\begin{aligned}\log 8 &= \log 2^3 \\ &= 3\log 2 \\ &= 3(0.301) \\ &= 0.903\end{aligned}$$

New: To solve an exponential equation, take the logarithm of each side.

Ex. 2 Solve each equation to 3 decimal places.

a) $2^x = 55$

Method 1 (from last day)

$\log_2 55 = x$

$\frac{\log 55}{\log 2} = x$

$x = 5.7813$

$\doteq 5.781$

Method 2 (New: Take the "log" of both sides)

$\log(2^x) = \log(55)$

$x\log 2 = \log 55$

$\frac{x\log 2}{\log 2} = \frac{\log 55}{\log 2}$

$x = \frac{\log 55}{\log 2}$

$= 5.781$

5.7813

b) $5^x = 20$

$\log 5^x = \log 20$

$x \log 5 = \log 20$

$x = \frac{\log 20}{\log 5}$

$\doteq 1.8613$

$\doteq 1.861$

1.8613

c) $3^{2x+1} = 14$

$\log 3^{2x+1} = \log 14$

$\frac{(2x+1)\log 3}{\log 3} = \frac{\log 14}{\log 3}$

$2x+1 = \frac{\log 14}{\log 3}$

$2x = \frac{\log 14}{\log 3} - 1$

$x = \frac{\left(\frac{\log 14}{\log 3} - 1\right)}{2}$

$\doteq 0.7010$

$= 0.701$

(Be careful of Bad Form)

Bad Form

$2x + 1\log 3 = \log 14$

$2x = \log 14 - 1\log 3$

Need Brackets

~~1.9021~~

0.7010

0.701

Law of Logarithms for Multiplication

$[x>0, y>0, a>0, a\neq 1]$

$$\log_a xy = \log_a x + \log_a y$$

Law of Logarithms for Division

$[x>0, y>0, a>0, a\neq 1]$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Ex. 3 [from 1.4.1 Ex. 2b] Suppose you invest \$1000 at 8% per year, compounded *quarterly*.
b) Estimate how many years it takes for the investment to grow to \$2800.

$$2800 = 1000(1.02)^{4x}$$

Method 1 (take the log of both sides immediately)

$\log[2800] = \log[1000(1.02)^{4x}]$

$\log 2800 = \log 1000 + \log(1.02)^{4x}$

$\log 2800 - \log 1000 = 4x \log 1.02$

$4\log 1.02 \qquad 4\log 1.02$

$\frac{(\log 2800 - \log 1000)}{(4\log 1.02)} = x$

$x \doteq 12.998$

$x \doteq 13$ years

12.99

Method 2 (Isolate the "exponential" first)

$\frac{2800}{1000} = (1.02)^{4x}$

$\log\left(\frac{2800}{1000}\right) = 4x \log 1.02$

$\frac{\log\left(\frac{2800}{1000}\right)}{4\log 1.02} = x$

$x \doteq 12.998$

$\doteq 13$ years

p.344 #9 and Worksheet 1.8.2

1.8.2: Logarithmic Functions Worksheet

Date: _____

1. Evaluate each of the following.

a) log 100 b) log 0.01 c) log 100 000

2. Use your calculator to evaluate each of the following to three decimal places.

a) log 25 b) log 0.004 c) log 636

3. Write in exponential form.

a) log 10 000 = 4 b) log 10 = 1 c) log 0.000 1 = – 4

d) $\log_4 64 = 3$ e) $\log_6 \frac{1}{216} = -3$ f) $\log_3 2187 = 7$

4. Write each of the following in logarithmic form.

a) $3^4 = 81$ b) $4^{-2} = \frac{1}{16}$ c) $4^{\frac{-3}{2}} = \frac{1}{8}$

5. Solve for x (round to three decimal places where necessary) .

a) $\log x = -3$ b) $\log_x 49 = 2$ c) $5^x = 8$

d) $3^{x+2} = 5$ e) $\log_4 \frac{1}{64} = x$

Answers

1a) 2 b) –2 c) 5 2a) 1.398 b) –2.398 c) 2.803

3a) $10^4 = 10\,000$ b) $10^1 = 10$ c) $10^{-4} = 0.0001$ d) $4^3 = 64$ e) $6^{-3} = \frac{1}{216}$ f) $3^7 = 2187$

4a) $\log_3 81 = 4$ b) $\log_4 \frac{1}{16} = -2$ c) $\log_4 \frac{1}{8} = \frac{-3}{2}$