Today

pp.116-117 #2, 3, 5*an estimate is required only, 6a*find the quadratic equation first then use the preceding interval method, 8, 9, 10, 11ab*use first principles, 13

p.118 (45 minutes max) #1,2,3,4a* use first principles

Optional Extra Practice Worksheet in Google Classroom

Recent Homework

Thurs. Sept. 20

p.86 #2bc, 4a

Use the "FIRST PRINCIPLES" FOR ALL RATE OF CHANGE CALCS

pp.86-89 #4c, 5, 10* do not approximate π +

Challenge: For the function $y = \frac{1}{x}$ find the exact rate of change at x = 2.

Mon. Sept. 24

4, 5

ENTERTAINMENT: pp.103-106 #1, 2*, 3 to 9*, 10, 11, 14

* in #2, the answer in the back has a small error. Do you know what it is?

Also, the answer for #9 in the back has some mistakes.

p.86 #10 *(Last year's version on next slide)*

10. To make a snow person, snow is being rolled into the shape of a sphere. The volume of a sphere is given by the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in centimetres. Use two different methods to estimate the instantaneous rate of change in the volume of the snowball with respect to the radius when $r = 5$ cm.

$$\text{iroc} = m_{\text{tangent}}$$
$$= \frac{V(r+h) - V(r)}{h}, \text{ as } h \to 0$$
$$= \frac{V(5+h) - V(5)}{h}, \text{ as } h \to 0$$
$$= \frac{\frac{4}{3}\pi(5+h)^3 - \frac{4}{3}\pi(5)^3}{h}, h \to 0$$
$$= \frac{\frac{4}{3}\pi\left[(5+h)^3 - (5)^3\right]}{h}, h \to 0$$
$$= \frac{\frac{4}{3}\pi\left[125 + 75h + 15h^2 + h^3 - 125\right]}{h}, h \to 0$$
$$= \frac{\frac{4}{3}\pi\left[h^3 + 15h^2 + 75h\right]}{h}, h \to 0$$
$$= \frac{\frac{4}{3}\pi \cancel{h}(h^2 + 15h + 75)}{\cancel{h}}, \text{ as } h \to 0$$
$$= \frac{4}{3}\pi(h^2 + 15h + 75), h \to 0$$
$$= \frac{4}{3}\pi(0 + 0 + 75)$$
$$= \frac{4}{\cancel{3}}\pi(\cancel{75}^{25})$$
$$= 100\pi \text{ cm}^3/\text{cm}$$

$(5+h)^3$ $\boxed{1\ 3\ 3\ 1}$

$= 5^3 + 3(5)^2h + 3(5)h^2 + h^3$

$= 125 + 75h + 15h^2 + h^3$

p.86 #10

This version of the solution does not common factor the 4/3 π first.

10. To make a snow person, snow is being rolled into the shape of a sphere. The volume of a sphere is given by the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in centimetres. Use two different methods to estimate the instantaneous rate of change in the volume of the snowball with respect to the radius when $r = 5$ cm.

$$\text{iroc} = \frac{f(x+h)-f(x)}{h},\ h\to 0$$

$$= \frac{f(5+h)-f(5)}{h},\ h\to 0$$

$$= \frac{\frac{4}{3}\pi(5+h)^3 - \frac{4}{3}\pi(5)^3}{h},\ h\to 0$$

$$= \frac{\frac{4}{3}\pi\left(5^3+3(5)^2h+3(5)h^2+h^3\right) - \frac{4}{3}\pi(5)^3}{h},\ h\to 0$$

$$= \frac{\frac{4}{3}\pi\left(125+75h+15h^2+h^3\right) - \frac{4}{3}\pi(125)}{h},\ h\to 0$$

$$= \frac{\cancel{\frac{4}{3}\pi(125)} + 100\pi h + 20\pi h^2 + \frac{4}{3}\pi h^3 \cancel{- \frac{4}{3}\pi(125)}}{h},\ h\to 0$$

$$= \frac{\cancel{h}(100\pi + 20\pi h + \frac{4}{3}\pi h^2}{\cancel{h}},\ h\to 0$$

$$= 100\pi + 20\pi h + \frac{4}{3}\pi h^2,\ h\to 0$$

$$= 100\pi \text{ cm}^3/\text{cm}$$

2.2_2 Challenge Question:

For the function $y = \dfrac{1}{x}$ find the exact rate of change at $x = 2$.

$$\text{iroc} = \frac{f(x+h)-f(x)}{h}, \text{ as } h \to 0$$

$$= \frac{f(2+h)-f(2)}{h}, \text{ as } h \to 0$$

$$= \frac{\frac{1}{(2+h)} - \frac{1}{2}}{h}, \; h \to 0$$

$$= \left(\frac{1}{2+h} - \frac{1}{2}\right) \times \frac{1}{h}, \; h \to 0$$

$$\frac{x+y}{h}$$

$$= (x+y) \div h$$

$$= (x+y) \times \frac{1}{h}$$

$$\frac{1}{2+h} - \frac{1}{2} \quad \text{LCD} = 2(2+h)$$

$$= \frac{1(2)}{(2+h)2} -$$

$$= \left[\frac{1(2)}{(2+h)2} - \frac{1(2+h)}{(2+h)(2)}\right] \times \frac{1}{h}, \; h \to 0$$

$$= \left[\frac{2}{2(2+h)} - \frac{2+h}{2(2+h)}\right] \times \frac{1}{h}, \; h \to 0$$

$$-\frac{2+h}{-(2+h)}$$

$$= -2-h$$

$$= \left[\frac{2-2-h}{2(2+h)}\right] \times \frac{1}{h}, \text{ as } h \to 0$$

$$= \left[\frac{-\cancel{h}}{2(2+h)}\right] \times \frac{1}{\cancel{h}}, \text{ as } h \to 0$$

$$= \frac{-1}{2(2+h)}, \text{ as } h \to 0$$

$$= -\frac{1}{2(2+0)}$$

$$= -\frac{1}{4}$$

From Optional Extra Practice Worksheet in Google Classroom

PRACTICE: Slope of the Tangent - $f(a+h)$

1. Determine a simplified expression for the slope of the tangent for each of the following.

a) $f(x) = 2x^2 + x + 1$ b) $f(x) = -x^2 + 10$ c) $f(x) = x^3 - 4$

$$m_{tangent} = \frac{f(x+h)-f(x)}{h}, \text{ as } h \to 0$$

$$= \frac{\left[2(x+h)^2 + (x+h) + 1\right] - \left[2x^2 + x + 1\right]}{h}, \text{ as } h \to 0$$

$$= \frac{\left[2(x^2+2xh+h^2) + x + h + 1\right] - 2x^2 - x - 1}{h}, \; h \to 0$$

$$= \frac{2x^2 + 4xh + 2h^2 + x + h + \cancel{1} - 2x^2 - x - \cancel{1}}{h}, \; h \to 0$$

$$= \frac{4xh + 2h^2 + h}{h}, \; h \to 0$$

$$= \frac{\cancel{h}(4x + 2h + 1)}{\cancel{h}}, \text{ as } h \to 0$$

$$= 4x + 2h + 1, \; h \to 0$$

$$= 4x + 1$$

p.104 #4

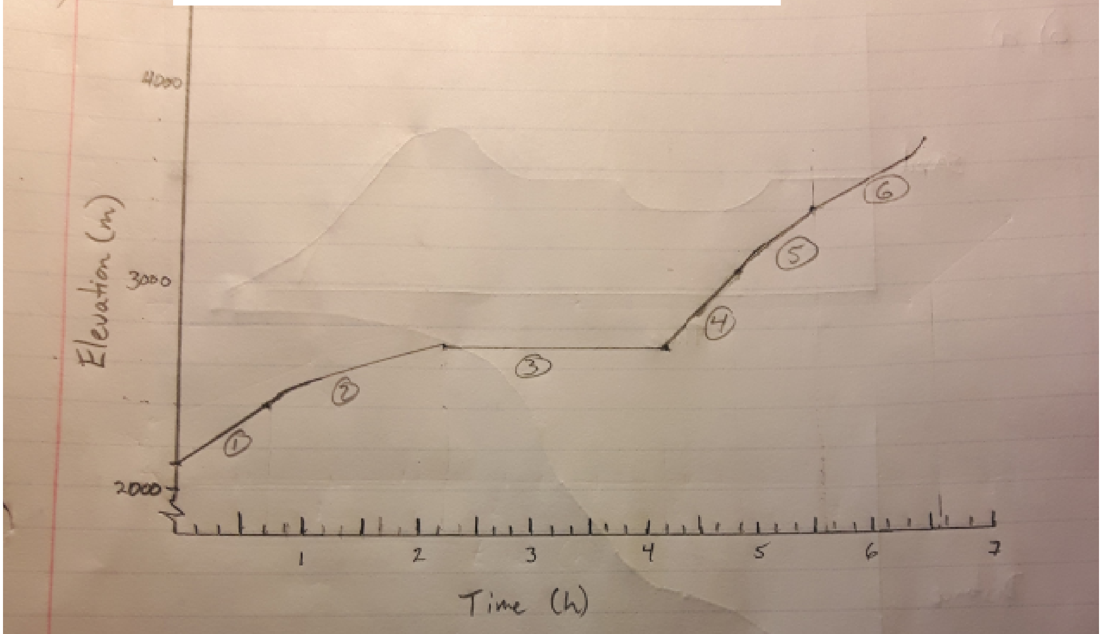
4. Rachel climbed Mt. Fuji while in Japan. There are 10 levels to the mountain. She was able to drive to Level 5, where she began her climb.
   - She walked at a constant rate for 40 min from Level 5 to Level 6.
   - She slowed slightly but then continued at a constant rate for a total of 90 min from Level 6 to Level 7.
   - She decided to eat and rest there, which took approximately 2 h.
   - From Level 7 to Level 8, a 40 min trip, she travelled at a constant rate.
   - Continuing on to Level 9, a 45 min trip, she decreased slightly to a new constant rate.
   - During most of the 1 h she took to reach Level 10, the top of Mt. Fuji, she maintained a constant rate. As she neared the top, however, she accelerated.

   a) Using the information given and the following table, draw an elevation versus time graph to describe Rachel's climb.

Level	5	6	7	8	9	10
Elevation (m)	2100	2400	2700	3100	3400	3740

   b) Calculate Rachel's average speed over each part of her climb.
   c) Draw a speed versus time graph to describe Rachel's climb.

4 a)



b)

① 5→6 $m = \frac{2400-2100}{40} = 7.5$ m/min

② 6→7 $m = \frac{2700-2400}{90} = 3.\bar{3}$ m/min

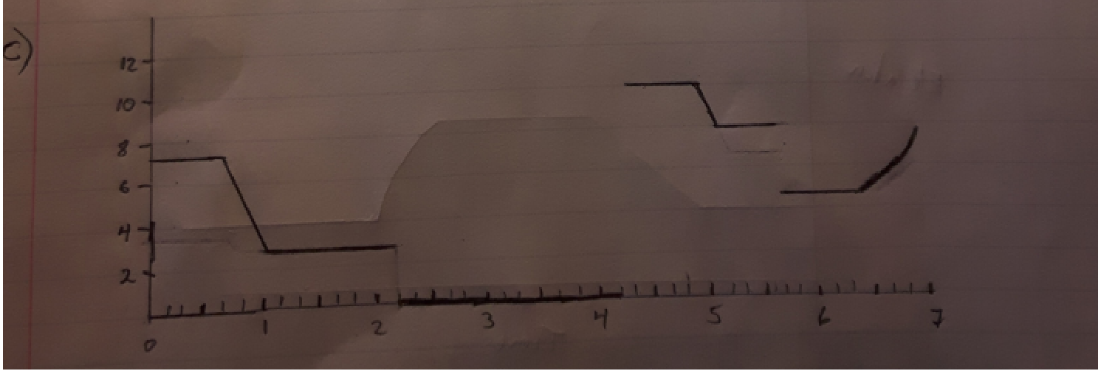
③ $m = 0$ (resting)

④ 7→8 $m = \frac{3100-2700}{40} \doteq 10$ m/min

⑤ $m = \frac{3400-3100}{45} = 6.\bar{6}$ m/min

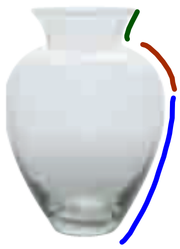
⑥ $m = \frac{3740-3400}{60} \doteq 5.\bar{6}$ m/min

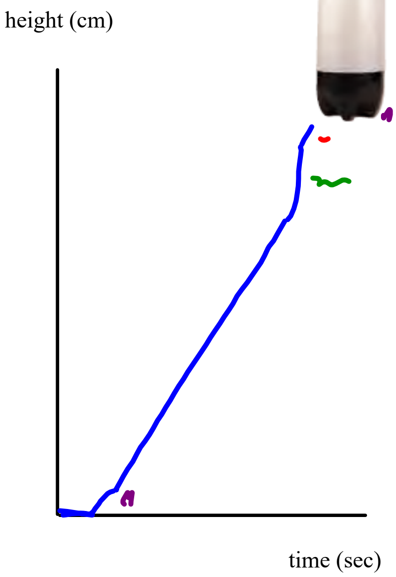
c)

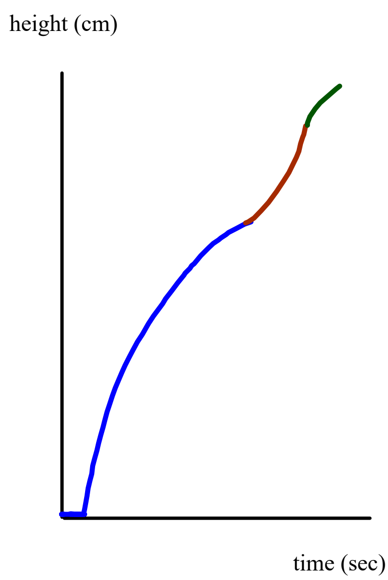


p.104 #5

5. The containers shown are being filled with water at a constant rate.
K Draw a graph of the water level versus time for each container.
a) a 2 L plastic pop bottle **b)** a vase



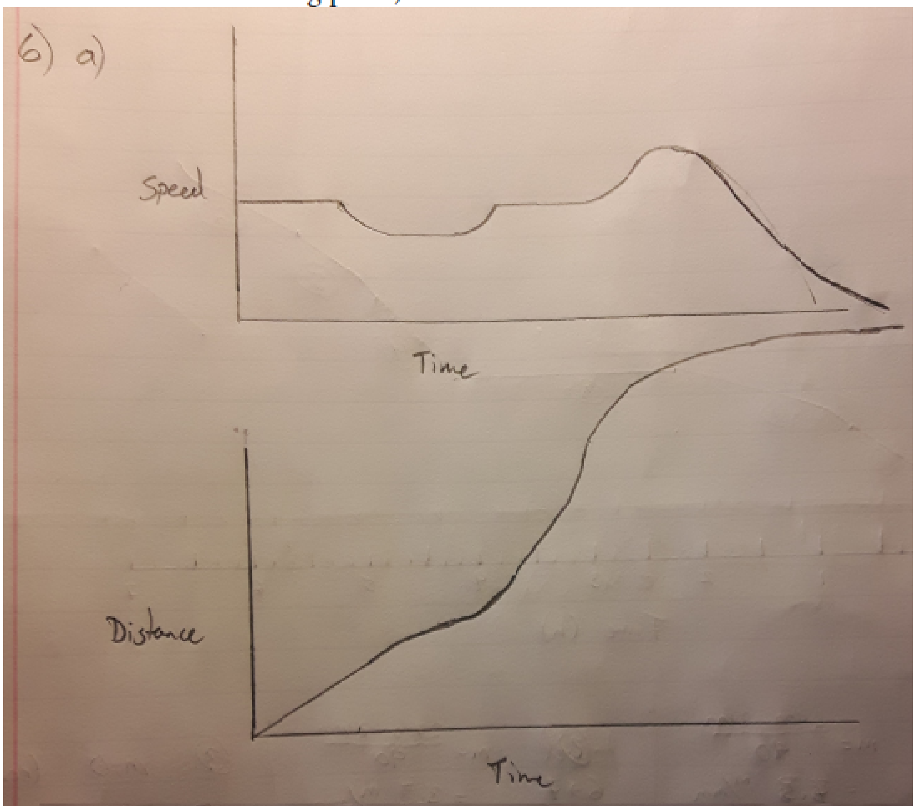


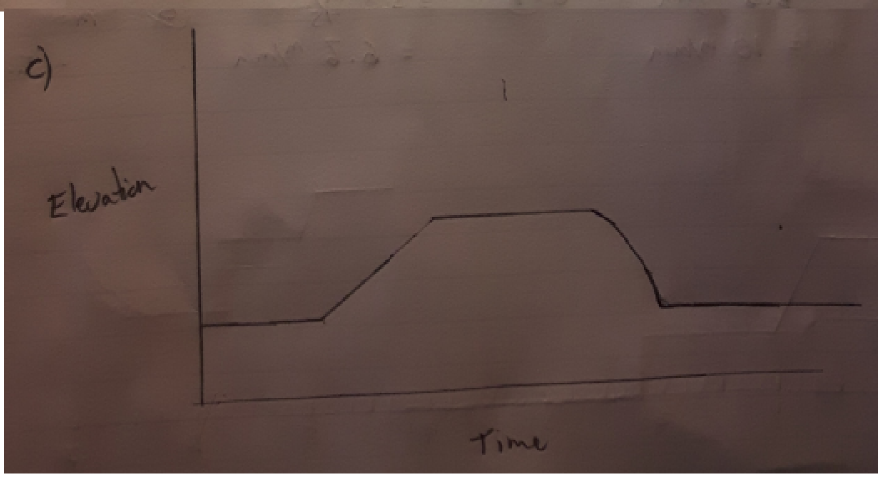


p.104 #6 **6.** John is riding a bicycle at a constant cruising speed along a flat road. He slows down as he climbs a hill. At the top of the hill, he speeds up, back to his constant cruising speed on a flat road. He then accelerates down the hill. He comes to another hill and coasts to a stop as he starts to climb.

a) Sketch a possible graph to show John's speed versus time, and another graph to show his distance travelled versus time.

b) Sketch a possible graph of John's elevation (in relation to his starting point) versus time.

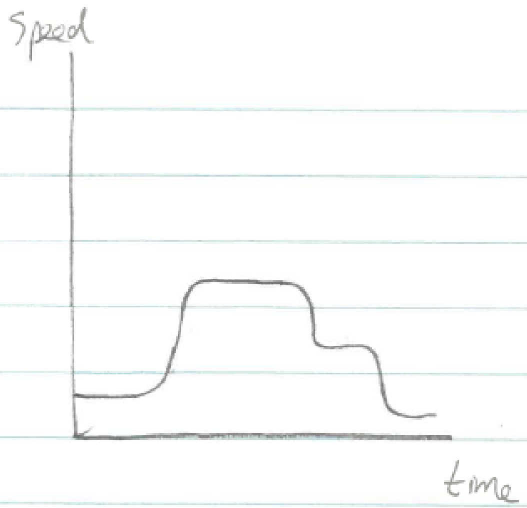




p.105 #9

9. A jockey is warming up a horse. Whenever the jockey has the horse accelerate or decelerate, she does so at a nonconstant rate—at first slowly and then more quickly. The jockey begins by having the horse trot around the track at a constant rate. She then increases the rate to a canter and allows the horse to canter at a constant rate for several laps. Next, she slowly begins to decrease the speed of the horse to a trot and then to a walk. To finish, the jockey walks the horse around the track once. Draw a speed versus time graph to represent this situation.

p.105 9.



p.106

11. A cross-country runner is training for a marathon. His training program requires him to run at different speeds for different lengths of time. His program also requires him to accelerate and decelerate at a constant rate. Today he begins by jogging for 10 min at a rate of 5 miles per hour. He then spends 1 min accelerating to a rate of 10 miles per hour. He stays at this rate for 5 min. He then decelerates for 1 min to a rate of 7 miles per hour. He stays at this rate for 30 min. Finally, to cool down, he decelerates for 2 min to a rate of 3 miles per hour. He stays at this rate for a final 10 min and then stops.

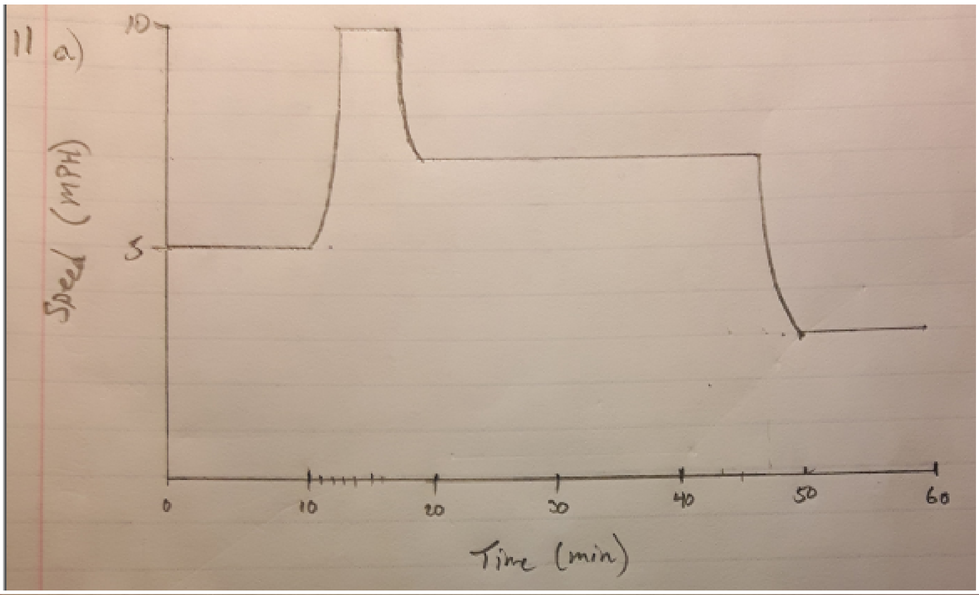
a) Make a speed versus time graph to represent this situation.

b) What is the instantaneous rate of change in the runner's speed at 10.5 min?

c) Calculate the runner's average rate at which he changed speeds from minute 11 to minute 49.

d) Explain why your answer for part c) does not accurately represent the runner's training schedule from minute 11 to minute 49.

11 a)



b) $m = \frac{10-5}{11-10}$

$= \frac{5}{1} = 5 \text{ mPH/min}$

c) $m = \frac{3-10}{49-11}$

$= \frac{-7}{38} \doteq -0.18 \frac{\text{mPH}}{\text{min}}$

d) This is an average over a long period of time. The runner decreased 2 times and held a constant rate in between the two extreme values (10 vs 3) for 30 minutes in the middle!