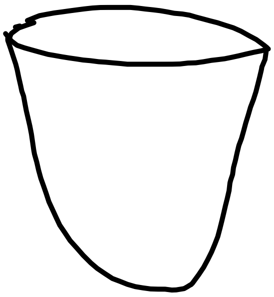
Yesterday

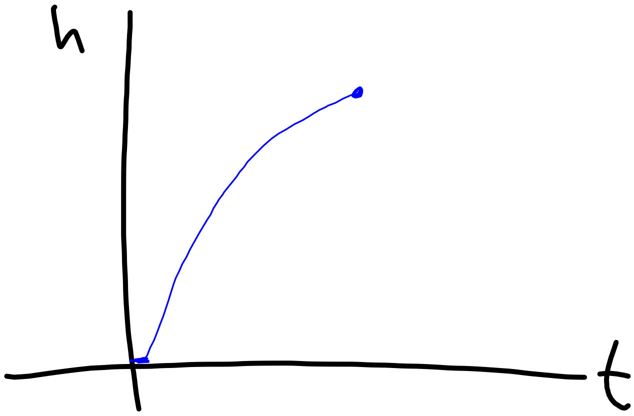
pp.116-117 #2, 3, 5*an estimate is required only, 6a*find the quadratic equation first then use the preceding interval method, 8, 9, 10, 11ab*use first principles, 13

p.118 (45 minutes max) #1,2,3,4a* use first principles

Optional Extra Practice Worksheet in Google Classroom p.117 Lesson 2.4

8. A sculptor makes a vase for flowers. The radius and circumference of the vase increase as the height of the vase increases. The vase is filled with water. Draw a possible graph of the height of the water as time increases.





p.117 **11.** A maximum or minimum is given for each of the following functions. Select a strategy, and verify whether the point given is a maximum or a minimum.

a) $f(x) = x^2 - 10x + 7$; $(5, -18)$

b) $g(x) = -x^2 - 6x - 4$; $(-3, 5)$

$$m_{tangent} = \frac{f(5+h) - f(5)}{h}, \ h \to 0$$

$$= \frac{(5+h)^2 - 10(5+h) + 7 - (5^2 - 10(5) + 7)}{h}, \text{ as } h \to 0$$

$$= \frac{\cancel{25} + \cancel{10h} + h^2 \cancel{- 50} \cancel{- 10h} \cancel{+ 7} - (\cancel{25} \cancel{- 50} \cancel{+ 7})}{h}, \ h \to 0$$

$$= \frac{h^2}{h}, \ h \to 0$$

$$= h, \ h \to 0$$

$$= 0$$

$\therefore$ at $x = 5$, local extrema

$h<0$	$h=0$	$h>0$
$-$	0	$+$
\	—	/

$\therefore (5, -18)$ is a local minimum.

p.117 **11.** A maximum or minimum is given for each of the following functions. Select a strategy, and verify whether the point given is a maximum or a minimum.

a) $f(x) = x^2 - 10x + 7$; $(5, -18)$

b) $g(x) = -x^2 - 6x - 4$; $(-3, 5)$

b) $m_{tangent} = \dfrac{g(x+h)-g(x)}{h}$, as $h \to 0$

$= \dfrac{g(-3+h)-g(-3)}{h}$, as $h \to 0$

$= \dfrac{\left[-(-3+h)^2-6(-3+h)-4\right]-\left[-(-3)^2-6(-3)-4\right]}{h}$, as $h \to 0$

$= \dfrac{\left[-(9-6h+h^2)+18-6h-4\right]-\left[\overbrace{-(9)+18-4}^{5}\right]}{h}$, $h \to 0$

$= \dfrac{\cancel{-9}+\cancel{6h}-h^2-\cancel{6h}+\cancel{14}\cancel{-5}}{h}$, $h \to 0$

$= \dfrac{-h^2}{h}$, $h \to 0$

$= -h$, $h \to 0$

$= 0$

$h<0$	$h=0$	$h>0$
$+$	0	$-$

$\therefore (-3,5)$ is a local maximum.

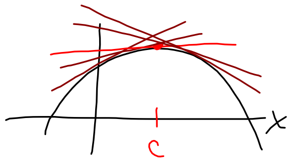
2.5 Solving Problems Involving Rates of Change

Math Learning Target:

"I know...

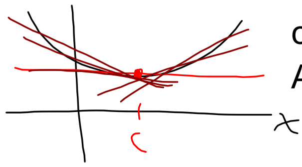
- what a global maximum and a global minimum is;
- what a local maximum and a local minimum is;
- when it is required to calculate a rate of change;
- how to calculate the exact rate of change."

Local Maximum at $x = c$



Given $y = f(x)$. For values of x near c ...
A **local maximum** at $x = c$ exists if the function's rate of change changes from positive to negative through $x = c$.
At $x = c$ the rate of change will be zero (or undefined).

Local Minimum at $x = c$



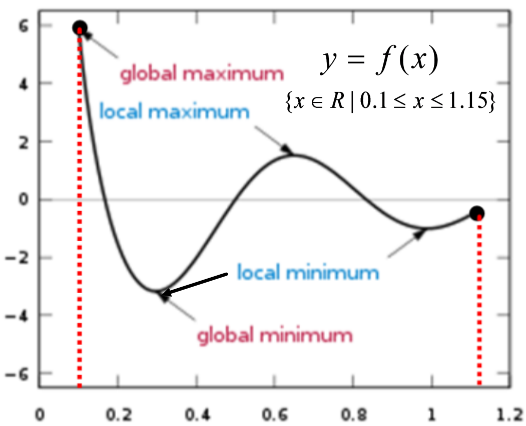
Given $y = f(x)$. For values of x near c ...
A **local minimum** at $x = c$ exists if the function's rate of change changes from negative to positive through $x = c$.
At $x = c$ the rate of change will be zero (or undefined).

Global Maximum at $x = c$

Given $y = f(x)$. A **global maximum** at $x = c$ exists if $f(c) > f(x)$ for all values of x in the function's domain.

Global Minimum at $x = c$

Given $y = f(x)$. A **global minimum** at $x = c$ exists if $f(c) < f(x)$ for all values of x in the function's domain.



Ex.1 Using first principles, prove that a local minimum value occurs at $x = 3$ for the function $f(x) = x^2 - 6x + 5$. Verify graphically.

$$m_{tangent} = \frac{f(x+h)-f(x)}{h}, \ h \to 0$$

$$= \frac{f(3+h)-f(3)}{h}, \ h \to 0$$

$$= \frac{(3+h)^2 - 6(3+h) + 5 - ((3)^2 - 6(3) + 5)}{h}, \text{ as } h \to 0$$

$$= \frac{9 + \cancel{6h} + h^2 - 18 - \cancel{6h} + 5 - (9 - 18 + 5)}{h}, \text{ as } h \to 0$$

$$= \frac{h^2}{h}, \text{ as } h \to 0$$

$$= h, \text{ as } h \to 0$$

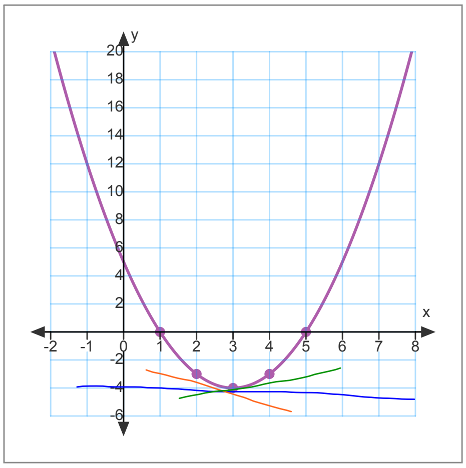
$$= 0$$

$\therefore$ at $x = 3$, it is a local extrema

$h<0$	$h=0$	$h>0$
$-$	0	$+$

$\because$ the $m_{tangent}$ goes from negative to positive,

$\therefore$ it is a local minimum.



$y = x^2 - 6x + 5$

Use FIRST PRINCIPLES FOR ALL RATE OF CHANGE CALCULATIONS

pp. 111-113 #1, 3, 4, 6c, 9a, 10, 14

$y = x^2 - 6x + 9 - 9 + 5$

$= (x-3)^2 - 4$

$y = (x-5)(x-1)$

AofS: $x = \frac{5+1}{2}$

$= 3$

$\therefore f(3) = 3^2 - 6(3) + 5$

$= -4$

$\therefore V(3, -4)$