3.1 Exploring Polynomial Functions

Math Learning Target:

"I can identify a polynomial function.
I can classify polynomial functions."

With a partner, complete: "***EXPLORE*** *the Math*" pp. 124-126, but only A, B, C, D, E, F**, G***see below, K, L.

All written work is to be written on a new piece of paper in your notebook.

desmos will be used for questions that require a "graphing calculator".
(Remember to sign in, name and then save your work, i.e. MHF 3.1 Exploring)

** *Recall:* finite differences are first differences, second differences, and so on...
The final answers for F are included on the next page of this document to save you work,
BUT YOU MUST UNDERSTAND HOW THEY WERE FOUND!!

***For question G use:
1) $f(x) = x^3 + 2x^2 - 3x - 2$
2) $f(x) = x^4 + 3x^3 + x^2 - 2x - 2$
3) $f(x) = x^5 - x^4 - 3x^3 + x^2 + x - 2$

Now **READ** p. 126 "**In Summary** ".

Finally, complete pp. 127-128 # 1, 2, 3d, 5, 7, 8

A **polynomial expression** (or **polynomial**) is a series of terms (added/subtracted) where each term is the product of a constant and a power of x with exponents that are non-negative integers.

A **polynomial function** is of the form:
$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$, where a_0, a_1, a_2 etc... are real number constants, and n is a non-negative integer.
Its finite differences eventually become constant.

The **degree** of a polynomial function will always be the value n as long as leading coefficient $a_n \neq 0$.

Study this answer for "F":

F.

$f(x) = x$	Δ_1
$f(-3) = -3$	
	1
$f(-2) = -2$	
	1
$f(-1) = -1$	
	1
$f(0) = 0$	
	1
$f(1) = 1$	
	1
$f(2) = 2$	
	1
$f(3) = 3$	

$f(x) = x^2$	Δ_1	Δ_2
$f(-3) = 9$		
	−5	
$f(-2) = 4$		2
	−3	
$f(-1) = 1$		2
	−1	
$f(0) = 0$		2
	1	
$f(1) = 1$		2
	3	
$f(2) = 4$		2
	5	
$f(3) = 9$		

$f(x) = x^3$	Δ_1	Δ_2	Δ_3
$f(-3) = -27$			
	19		
$f(-2) = -8$		−12	
	7		6
$f(-1) = -1$		−6	
	1		6
$f(0) = 0$		0	
	1		6
$f(1) = 1$		6	
	7		6
$f(2) = 8$		12	
	19		
$f(3) = 27$			

$f(x) = x^4$	Δ_1	Δ_2	Δ_3	Δ_4
$f(-3) = 81$				
	−65			
$f(-2) = 16$		50		
	−15		−36	
$f(-1) = 1$		14		24
	−1		−12	
$f(0) = 0$		2		24
	1		12	
$f(1) = 1$		14		24
	15		36	
$f(2) = 16$		50		
	65			
$f(3) = 81$				

$f(x) = x^5$	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$f(-3) = -243$					
	211				
$f(-2) = -32$		−180			
	31		150		
$f(-1) = -1$		−30		−120	
	1		30		120
$f(0) = 0$		2		0	
	1		30		120
$f(1) = 1$		30		120	
	31		150		
$f(2) = 32$		180			
	211				
$f(3) = 243$					

p. 127 **7.** Create equations for a linear, a quadratic, a cubic, and a quartic polynomial function that all share the same *y*-intercept of 5.

$y = 2x + 5$

$y = x^2 - 6x + 5$

$y = 3x^3 - 6x^2 + 2x + 5$

$y = -\frac{1}{4}x^4 - 6x^3 + 2x^2 + 5$

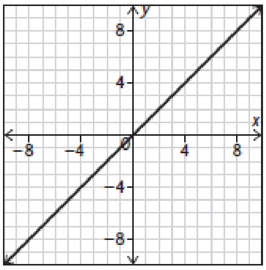
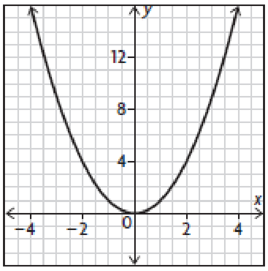
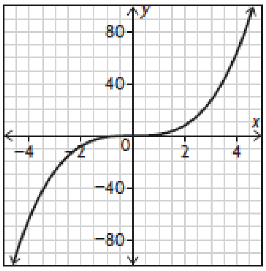
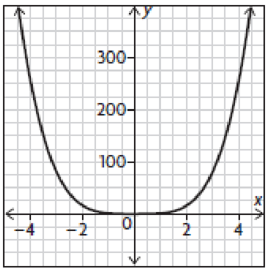
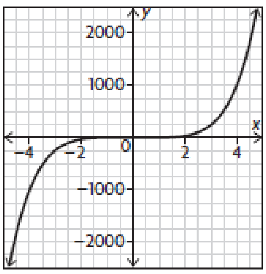
Answers to Explore the Math

A. The polynomial expressions all involve only the sum of constant multiples of non-negative integer powers of *x*.

B. Some have negative or fractional powers of *x*. Others have an *x* in the denominator of a fraction. Still others have *x*'s and *y*'s, and one even involves a sine function.

C. Answers may vary. A polynomial expression is an expression that is the sum of constant multiples of non-negative integer powers of *x*.

D.

Polynomial Function	Type	Sketch of Graph	Description of Graph	Domain and Range	Existence of Asymptotes?
$f(x) = x$	linear		The graph is a line through (0, 0) at a $45°$ angle to the x-axis.	$D = \{x \in \mathbf{R}\}$ $R = \{f(x) \in \mathbf{R}\}$	None
$f(x) = x^2$	quadratic		The graph is a parabola through (0, 0) opening up.	$D = \{x \in \mathbf{R}\}$ $R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	None
$f(x) = x^3$	cubic		The graph increases from $-\infty$ to ∞ and flattens out around the origin.	$D = \{x \in \mathbf{R}\}$ $R = \{f(x) \in \mathbf{R}\}$	None
$f(x) = x^4$	quartic		The graph looks like the graph of x^2 only much flatter near the origin.	$D = \{x \in \mathbf{R}\}$ $R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	None
$f(x) = x^5$	quintic		The graph looks like the graph of x^3 only much flatter near the origin.	$D = \{x \in \mathbf{R}\}$ $R = \{f(x) \in \mathbf{R}\}$	None

E. The linear, cubic, and quintic behave similarly and all have odd powers, while the quadratic and quartic behave similarly and have even powers.

F.

$f(x) = x$	Δ_1
$f(-3) = -3$	
	1
$f(-2) = -2$	
	1
$f(-1) = -1$	
	1
$f(0) = 0$	
	1
$f(1) = 1$	
	1
$f(2) = 2$	
	1
$f(3) = 3$	

$f(x) = x^2$	Δ_1	Δ_2
$f(-3) = 9$		
	−5	
$f(-2) = 4$		2
	−3	
$f(-1) = 1$		2
	−1	
$f(0) = 0$		2
	1	
$f(1) = 1$		2
	3	
$f(2) = 4$		2
	5	
$f(3) = 9$		

$f(x) = x^3$	Δ_1	Δ_2	Δ_3
$f(-3) = -27$			
	19		
$f(-2) = -8$		−12	
	7		6
$f(-1) = -1$		−6	
	1		6
$f(0) = 0$		0	
	1		6
$f(1) = 1$		6	
	7		6
$f(2) = 8$		12	
	19		
$f(3) = 27$			

$f(x) = x^4$	Δ_1	Δ_2	Δ_3	Δ_4
$f(-3) = 81$				
	−65			
$f(-2) = 16$		50		
	−15		−36	
$f(-1) = 1$		14		24
	−1		−12	
$f(0) = 0$		2		24
	1		12	
$f(1) = 1$		14		24
	15		36	
$f(2) = 16$		50		
	65			
$f(3) = 81$				

$f(x) = x^5$	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
$f(-3) = -243$					
	211				
$f(-2) = -32$		−180			
	31		150		
$f(-1) = -1$		−30		−120	
	1		30		120
$f(0) = 0$		2		0	
	1		30		120
$f(1) = 1$		30		120	
	31		150		
$f(2) = 32$		180			
	211				
$f(3) = 243$					

The number of finite differences to get down to a constant is determined by the degree of the polynomial.

G. Answers may vary, but the number of finite differences needed to obtain a constant should be the same as the degree of the polynomial.

Answers to Reflecting

J. There is a y or a $f(x)$ present.

K. Both are sums of constant multiples of non-negative integer powers of x.

L. As the degree increases, the graph flattens out near the origin and becomes steeper away from the origin. The number of finite differences needed to obtain a constant increases with the degree.

M. The definition of a polynomial function can now be "a function in which finite differences eventually become constant."