

**Before we begin, are there any questions from last day's work?**

pp.217-218      1, 2c, 3d, 4b, 6, 7

***SWYK 2.1 is first.***

**Today's Learning Goal(s):**

By the end of the class, I will be able to:

- a) use a quadratic model to solve a problem with ***and without*** technology.

## 2.8.1 Modeling using Quadratic Functions

Date: Oct. 4/18

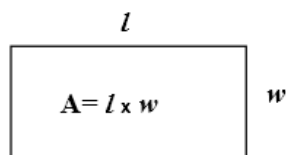
Ex.1

Sixteen metres of fencing are available to enclose a rectangular garden.

- Represent the area of the garden as a function of the length of one side.
- Graph the function.
- What dimensions provide an area greater than  $12 \text{ m}^2$ ?

Solution

- Let  $w$  represent the width of the garden in m.  
Let  $l$  represent the length of the garden in m.



$$P = 2l + 2w$$

$$16 = 2l + 2w$$

$$8 = l + w$$

$$8 - w = l$$

$$\text{Since } A = l w$$

$$A = (8 - w)w$$

$$\begin{aligned} A &= (8 - w)w \\ &= 8w - w^2 \\ &= -w^2 + 8w \\ &\therefore a = -1 \end{aligned}$$

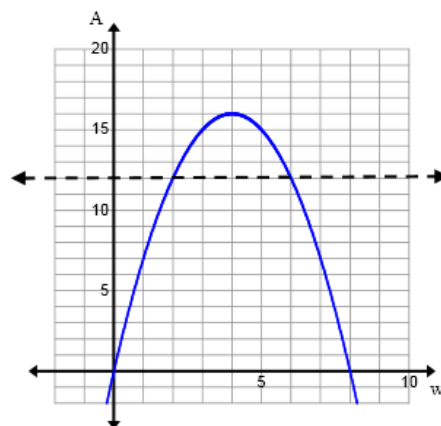
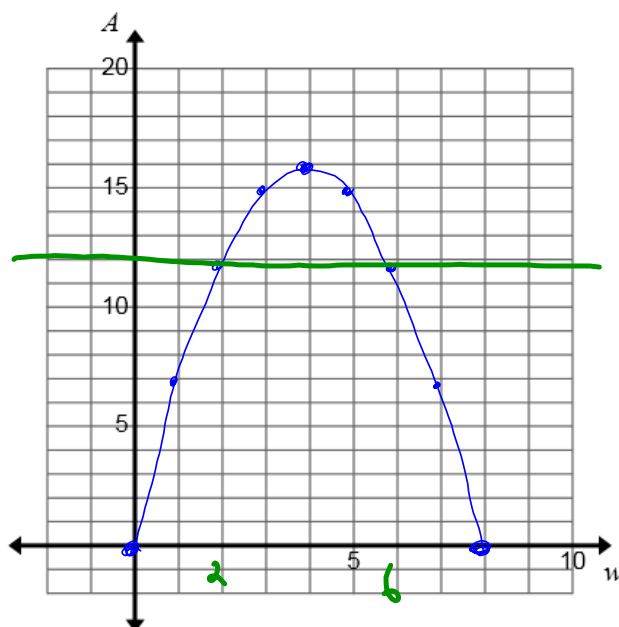
b)

the zeros (x-intercepts) are 0 and 8

Find the vertex half way between the zeros,

or complete the square to get  $A = -1(w - 4)^2 + 16$ 

$$\begin{aligned} \text{Avg: } w &= \frac{0+8}{2} \rightarrow A = (8-w)w \\ w &= 4 & A &= (8-4)(4) \\ & & &= 16 \\ A &= -w^2 + 8w \\ &= -1(w^2 - 8w + 16 - 16) \\ &= -1(w-4)^2 + 16 \therefore V(4, 16) \end{aligned}$$



- Draw in the horizontal line  $y = 12$ .

The intersection points represent the width of the garden when the area is  $12 \text{ m}^2$ . $\therefore$  if the width is between (but NOT INCLUDING) 2 and 6 m,the dimensions provide an area greater than  $12 \text{ m}^2$ .This is written  $2 < w < 6$ 

if c) asked  $< 12 \text{ m}^2$   
 $\therefore 0 \leq w < 2$  and  $6 < w \leq 8$

## 2.8.1 Modeling using Quadratic Functions

Ex. 2

When bicycles are sold for \$300 each, a cycle store can sell 160 in a season.

For every \$25 increase in the price, the number sold drops by 10.

- Represent the sales revenue as a function of the price.
- Use a graphing calculator to graph the function.
- How many bicycles were sold when the total sales revenue is \$33 000?  
What is the price of one bicycle?
- What range of prices will give sales revenue that exceeds \$40 000?

Solution

- The quantities that vary all need to be defined (as variables).

Let  $p$  represent the selling price, in dollars.Let  $n$  represent the number of bicycles sold.Let  $R$  represent the revenue, in dollars.

$$\text{Revenue} = \underset{p}{(\text{price of a bicycle})} \times \underset{\substack{\text{x (needs to be represented as a function of price)}}}{(\text{number of bicycles sold})}$$

(This is the hardest part of this problem.)

Rough work:

- the price increase =  $p - 300$

Check: If the new price is \$375,  
then the price increase = 
$$\begin{aligned} p - 300 &= 375 - 300 \\ &= 75 \end{aligned}$$

- the number of \$25 increases =  $\frac{p - 300}{25}$

Check: If the new price is \$375,  
then the number of \$25 increases = 
$$\begin{aligned} \frac{375 - 300}{25} &= \frac{75}{25} \\ &= 3 \text{ increases of } \$25 \end{aligned}$$

$$\begin{aligned} \text{iii) the number of bicycles sold} &= 160 - 10 \left( \frac{p - 300}{25} \right) \\ &= 160 - 2 \left( \frac{p - 300}{5} \right) \\ &= 160 - \frac{2}{5}(p - 300) \\ &= 160 - \frac{2}{5}p + 120 \\ &= -\frac{2}{5}p + 280 \end{aligned}$$

←Show Cancelling

$$\begin{aligned} &= -\frac{2}{5}(p - 300) \\ &= -\frac{2}{5}p - \frac{2}{5}(-300) \\ &= -\frac{2}{5}p - 2(-60) \end{aligned}$$

Now, Revenue = (price of a bicycle) x (number of bicycles sold)

$$= p \left( -\frac{2}{5}p + 280 \right)$$

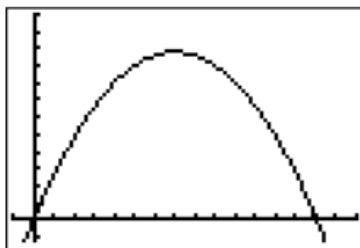
$$= -\frac{2}{5}p^2 + 280p$$

$$\text{or } (-0.4p^2 + 280p)$$

- b) Use a graphing calculator to graph the function.

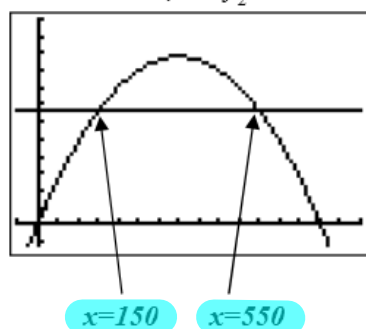
$$\text{let } y_1 = -0.4x^2 + 280x \quad \text{or} \quad y_1 = -\frac{2}{5}x^2 + 280x$$

☒ X-Axis      
 $-50 \leq x \leq 800$     Step: 50  
☒ Y-Axis      
 $-6000 \leq y \leq 60000$     Step: 5000



- c) How many bicycles were sold when the total sales revenue is \$33 000?  
What is the price of one bicycle?

If  $R = 33\,000$ , let  $y_2 = 33000$



Find the intersection points to represent the price of one bicycle when the revenue is \$33 000.

$$\therefore p = \mathbf{150} \quad \text{or} \quad p = \mathbf{550}$$

$\therefore$  the price of one bicycle is **\$150** or **\$550**

Recall: Revenue = (price of a bicycle)  $\times$  (number of bicycles sold)

$$\therefore \text{number of bicycles sold} = \frac{\text{Revenue}}{\text{price of a bicycle}}$$

$$\text{if } p = \mathbf{150}$$

$$\begin{aligned} \text{number of bicycles} &= \frac{33\,000}{150} \\ &= \mathbf{220} \end{aligned}$$

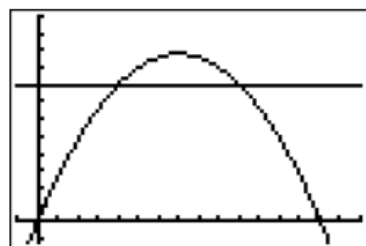
$$\text{or if } p = \mathbf{550}$$

$$\begin{aligned} \text{or number of bicycles} &= \frac{33\,000}{550} \\ &= \mathbf{60} \end{aligned}$$

$\therefore$  **60** bicycles were sold if the sales revenue is \$33 000.  
(since the price *increases* will result in lower sales)

- d) What range of prices will give sales revenue that exceeds \$40 000?

If  $R = 40\,000$ , let  $y_3 = 40000$



(Don't forget to "turn off"  $y_2$ )

Find the intersection points to represent the price of one bicycle when the revenue is exactly \$40 000.

$$\therefore p = \mathbf{200} \quad \text{or} \quad p = \mathbf{500}$$

Because we want when the revenue exceeds \$40 000,  
we DO NOT INCLUDE the intersection points in the solution.

$$\therefore \text{if } R > \$40\,000, \text{ then } \mathbf{200} < p < \mathbf{500}$$

**Assigned Work**

**pp. 224-225 #4(a-c), 6, 7, 10**