If the remainder is zero, then
we have factors of the dividend: divisor and quotient.

Alternate using Long Division and Synthetic Division...

3.5 Complete pp.168-170 #5cd, 7ad, 8bc, 9ab, 10ae, 11, 12, 15
Challenge yourself! #17, 18, 19

3.3 Now complete pp.146-148 #1, 2a, 4b, 6be, 8ab, 9ab, 10d, 13a, 16*
* for 16b you will need to use desmos

3.4 pp.155-158 #1, 2*, 3ab, 4bd, 5a, 6ab, 8**, 9af (*for #9 see Ex. 2 on p.153*), 10, 14

Today's Work

p. 122 #1d, 2

pp. 184-185 #1, 2*, 3, 4d, 5d, 6, 8bcd, 9cef, 10ad, 12cd
**the answer is wrong in the back for #2*

p.138 #16

16. a) Suppose that $f(x) = ax^2 + bx + c$. What must be true about the coefficients if f is an even function?

b) Suppose that $g(x) = ax^3 + bx^2 + cx + d$. What must be true about the coefficients if g is an odd function?

a) if even $f(-x) = f(x)$

* a must be positive (+ve)

x b must be negative

(c must be positive)

$\therefore b = 0$ is the only parameter.

$f(x) = +ax^2 - bx + c$

$f(-x) = +a(-x)^2 - b(-x) + c$

$= +a(+x^2) + bx + c$

$= ax^2 + bx + c$

b) Suppose that $g(x) = ax^3 + bx^2 + cx + d$. What must be true about the coefficients if g is an odd function?

$g(x) = ax^3 + bx^2 + cx + d$

if odd $f(-x) = -f(x)$

$g(-x) = a(-x)^3 + b(-x)^2 + c(-x) + d$

$= a(-x^3) + b(x^2) + c(-x) + d$

$= -ax^3 + bx^2 - cx + d$

$= -1(ax^3 - bx^2 + cx - d)$

Fail

$\therefore b + d$ are incorrect.

$\therefore b = 0$ and $d = 0$.

p.157 **8.** Dikembe has reflected the function $g(x) = x^3$ in the x-axis, vertically compressed it by a factor of $\frac{2}{3}$, horizontally translated it 13 units to the right, and vertically translated it 13 units down. Three points on the resulting curve are $\left(11, -\frac{23}{3}\right)$, $(13, -13)$, and $\left(15, -\frac{55}{3}\right)$. Determine the original coordinates of these three points on $g(x)$.

$(x, y) \rightarrow \left((x+13), \frac{2}{3}y - 13 \right)$ $\quad g(x) = \frac{2}{3}(x-13)^3 - 13$

Inverse: $(x, y) \rightarrow \left(x-13, \frac{3}{2}(y+13) \right)$

$\left(11, \frac{-23}{3}\right) \rightarrow \left(11-13, \frac{3}{2}\left(\frac{-23}{3} + 13\right) \right)$

$= \left(-2, \frac{3}{2}\left(\frac{-23}{3} + \frac{39}{3} \right) \right)$

$= \left(-2, \cancel{\frac{3}{2}}^{1} \left(\cancel{\frac{16}{3}}^{8} \right) \right)$

$= (-2, 8)$

p.169 #5d

d) $(x^4 + 3x^3 - 2x^2 + 5x - 1) \div (x^2 + 7)$

$$\begin{array}{r l}
 & x^2 + 3x - 9 \\
x^2+7 \; \big) & x^4 + 3x^3 - 2x^2 + 5x - 1 \\
 & x^4 \qquad + 7x^2 \\
\hline
 & 3x^3 - 9x^2 + 5x \\
- & 3x^3 \qquad + 21x \\
\hline
 & -9x^2 - 16x - 1 \\
+ & -9x^2 \qquad \pm 63 \\
\hline
 & -16x + 62
\end{array}$$

$$\therefore x^4+3x^3-2x^2+5x-1 = (x^2+7)(x^2+3x-9) + (-16x+62)$$

dividend = divisor x quotient + remainder

p.169 **7.** Each divisor was divided into another polynomial, resulting in the given quotient and remainder. Find the other polynomial (the dividend).

d) divisor: $x^2 + 7x - 2$, quotient: $x^4 + x^3 - 11x + 4$, remainder: $x^2 - x + 5$

dividend = divisor x quotient + remainder

$$= (x^2+7x-2)(x^4+x^3-11x+4) + (x^2-x+5)$$

$$= x^6 + x^5 - 11x^3 + 4x^2 + 7x^5 + 7x^4 - 77x^2 + 28x - 2x^4 - 2x^3 + 22x - 8x^2 - x + 5$$

$$= x^6 + 8x^5 + 5x^4 - 13x^3 - 72x^2 + 49x - 3$$

p.169

9. Each dividend was divided by another polynomial, resulting in the given quotient and remainder. Find the other polynomial (the divisor).

a) dividend: $5x^3 + x^2 + 3$, quotient: $5x^2 - 14x + 42$, remainder: -123

dividend = divisor × quotient + remainder

dividend − remainder = div. × quotient

$$\frac{\text{dividend} - \text{rem}}{\text{quotient}} = \text{divisor}$$

$$\frac{5x^3+x^2+3-(-123)}{5x^2-14x+42} = \text{divisor}$$

$$\text{divisor} = \frac{5x^3+x^2+126}{5x^2-14x+42}$$

$$\begin{array}{r} x + 3 \\ 5x^2-14x+42 \overline{)\, 5x^3+x^2+0x+126} \\ -\; 5x^3-14x^2+42x \\ \hline 15x^2-42x+126 \\ \underline{15x^2-42x+126} \\ 0R. \end{array}$$

$\therefore$ divisor $= x+3$

p.170

15. If the divisor of a polynomial, $f(x)$, is $x - 4$, then the quotient is $x^2 + x - 6$ and the remainder is 7.

a) Write the division statement.

b) Rewrite the division statement by factoring the quotient.

c) Graph $f(x)$ using your results in part b).

dividend = divisor × quotient + remainder

a) $f(x) = (x-4)(x^2+x-6) + 7$

b) $= (x-4)(x+3)(x-2) + 7$

let $g(x) = (x-4)(x+3)(x-2)$

y-int, let $x=0$

$g(0) = (-4)(3)(-2)$

$= 24$

$g(\ ) = (3-4)(3+3)(3-2)$

$= (-1)(6)(1)$

$= -6$