

3.6 Factoring Polynomials: Part 1



Math Learning Target:

"I can state the Remainder Theorem and the Factor Theorem.

By the end of next class, I can always apply it, when it is applicable."

Given: $p(x) = x^3 + 2x^2 - 5x - 6$

Evaluate: $p(-2)$

$$p(1) = (1)^3 + 2(1)^2 - 5(1) - 6$$

$$\begin{aligned} p(-2) &= (-2)^3 + 2(-2)^2 - 5(-2) - 6 \\ &= -8 + 8 + 10 - 6 \\ &= 4 \end{aligned}$$

$$= 1 + 2 - 5 - 6$$

$$= -8$$

Divide $p(x)$ by:

a) $x + 2$

$$\begin{array}{r} x^2 - 5 \\ \hline x+2 \sqrt{x^3 + 2x^2 - 5x - 6} \\ \quad x^3 + 2x^2 \\ \hline \quad 0 - 5x - 6 \\ \quad \quad - 5x + 10 \\ \hline \quad \quad \quad 4R \end{array}$$

b) $x - 1$

$$\begin{array}{r} 1 \quad 2 \quad -5 \quad -6 \\ \downarrow \quad \quad \quad \quad \\ 1 \quad 3 \quad -2 \\ \hline 1 \quad 3 \quad -2 \quad (-8) \\ \text{Remainder.} \end{array}$$

What do you notice?

- 👉 The remainder can be found easily,
without using long (or synthetic) division.

The Remainder Theorem

- 👉 Given polynomial function $p(x)$,
- 👉 If $p(x)$ is divided by $x - a$, where $a \in \mathbb{R}$, the remainder is $p(a)$.

For the same polynomial function $p(x) = x^3 + 2x^2 - 5x - 6$

a) Evaluate $p(2)$ b) Divide by $x - 2$ $\therefore k = 2$

$$\begin{aligned} p(2) &= (2)^3 + 2(2)^2 - 5(2) - 6 \\ &= 8 + 8 - 10 - 6 \\ &= 0 \end{aligned}$$

$$\begin{array}{r} 2 \Big| 1 \ 2 \ -5 \ -6 \\ \downarrow \ 2 \ 8 \ 6 \\ 1 \ 4 \ 3 \ 0 \end{array} \quad \text{is expected}$$

$\hookrightarrow x^2 + 4x + 3$

👉 Thus, when $p(x)$ is divided by $x - 2$, the remainder is zero.

Division statement :

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\begin{aligned} x^3 + 2x^2 - 5x - 6 &= (x-2)(x^2 + 4x + 3) + 0 \\ &= (x-2)(x+3)(x+1) \end{aligned}$$

The Factor Theoremiff \Leftrightarrow Given polynomial function $f(x)$,#1) If $f(a) = 0$, then $x - a$ is a factor of $f(x)$, and,#2) If $x - a$ is a factor, then $f(a) = 0$, for all $a \in \mathbb{R}$ Factor this polynomial completely, if it is factorable: $f(x) = x^3 - 6x^2 - x + 30$

$$\begin{aligned} f(1) &= 1 - 6 - 1 + 30 \\ &= 24 \end{aligned}$$

$$\begin{aligned} f(-1) &= -1 - 6 + 1 + 30 \\ &= 24 \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^3 - 6(2)^2 - (2) + 30 \\ &= 8 - 24 - 2 + 30 \\ &= 12 \end{aligned}$$

$$\begin{aligned} f(-2) &= (-2)^3 - 6(-2)^2 - (-2) + 30 \\ &= -8 - 24 + 2 + 30 \\ &= 0 \end{aligned}$$

$\therefore x+2$ is a factor

try $\pm (1, 2, 3, 5, 6, 10, 15, 30)$

$$\begin{array}{r} 1 - 6 - 1 \ 30 \\ \downarrow \quad -2 \quad 16 - 30 \\ 1 - 8 \ 15 \text{ OR } * \text{Expected} \end{array}$$

$$\text{let } g(x) = x^2 - 8x + 15$$

$$\therefore f(x) = (x+2)g(x) \quad \pm (1, 3, 5, 15)$$

$$f(x) = (x+2)(x-3)(x-5) \quad g(3) = (3)^2 - 8(3) + 15 \\ = 9 - 24 + 15$$

$$\therefore f(x) = (x+2)(x^2 - 8x + 15) = 0$$

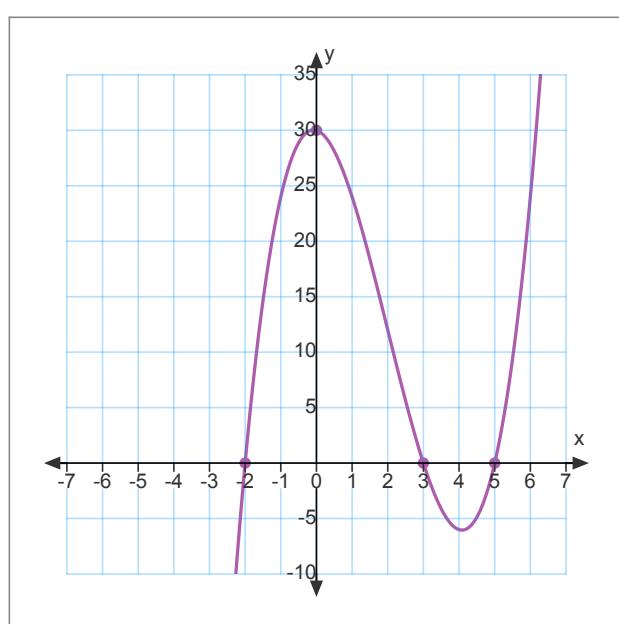
$$= (x+2)(x-3)(x-5) \quad 3 \left| \begin{array}{r} 1 - 8 \ 15 \\ \downarrow \quad 3 - 15 \\ 1 - 5 \text{ OR} \end{array} \right.$$

zeros: $-2, 3, 5$ order: $1, 1, 1$

$$\hookrightarrow x-5$$

$$f(0) = 30$$

$$y = x^3 - 6x^2 - x + 30$$



Today's entertainment
pp. 176-177 #1 to 3, 4ace, 5ace