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From yesterday's entertainment:
p.177 #6de, 7e, 8e, 9, 10, 14, 16, 17

7. Factor fully. 8. Use the factored form of f(x) to sketch the graph of each function.

e) $f(x) = x^3 - x^2 + x - 1$

7e) $= x^2(x-1) + 1(x-1)$

$0 = (x-1)(x^2+1)$

$x = 1$ or $x^2 + 1 = 0$

$x^2 = -1$

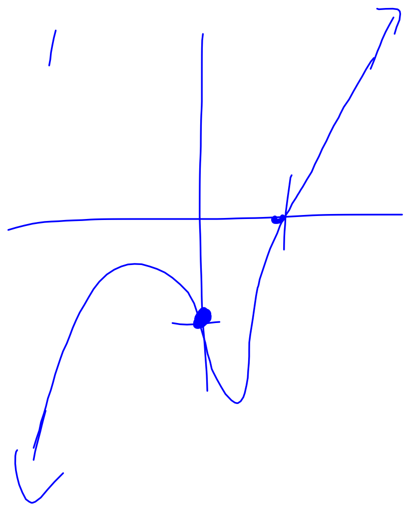
No Real Roots

8e) Zeros: 1

order: 1

$f(0) = -1$

y-int



9. The polynomial $12x^3 + kx^2 - x - 6$ has $2x - 1$ as one of its factors. Determine the value of k.

$2x - 1 = 0$

if $f(x) = 12x^3 + kx^2 - x - 6$ $\quad x = \frac{1}{2}$ $\quad \therefore f(\frac{1}{2}) = 0$

$f(\frac{1}{2}) = 12(\frac{1}{2})^3 + k(\frac{1}{2})^2 - (\frac{1}{2}) - 6$

$= \cancel{12}^{3}(\frac{1}{\cancel{8}_{2}}) + k(\frac{1}{4}) - \frac{1}{2} - 6$

$= \frac{3}{2} + \frac{k}{4} - \frac{1}{2} - \frac{12}{2}$

$0 = \frac{k}{4} + \frac{3-1-12}{2}$

$= \frac{k}{4} + \frac{-10}{2}$

$= \frac{k}{4} - 5$

$5 = \frac{k}{4}$

$\therefore k = 20$

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$\rightarrow f(1)=10 \;;\; f(2)=51$

10. When $ax^3 - x^2 + 2x + b$ is divided by $x - 1$, the remainder is 10. When it is divided by $x - 2$, the remainder is 51. Find a and b.

$f(x)=ax^3-x^2+2x+b$

$f(1)=a(1)^3-(1)^2+2(1)+b$

$=a-1+2+b$

$10=a+b+1$

$9=a+b$

$9-a=b$

$f(2)=a(2)^3-(2)^2+2(2)+b$

$=8a-4+4+b$

$51=8a+b$

$51=8a+(9-a)$

$51=8a+9-a$

$51-9=7a$

$42=7a$

$a=6$

$b=9-6$

$=3$

14. Show that $x - a$ is a factor of $x^4 - a^4$.

x^4-a^4

$=(x^2-a^2)(x^2+a^2)$

$=(x-a)(x+a)(x^2+a^2)$

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16. Use the factor theorem to prove that $x^2 - x - 2$ is a factor of $x^3 - 6x^2 + 3x + 10$.

$f(x) = x^3 - 6x^2 + 3x + 10$

$f(-1) = (-1)^3 - 6(1)^2 + 3(-1) + 10$

$= -1 - 6 - 3 + 10$

$= 0$

$\therefore (x+1)$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 3 & 10 \\ & \downarrow & -1 & 7 & -10 \\ \hline & 1 & -7 & 10 & 0R \end{array}$$

$x^2 - 7x + 10$

$= (x-5)(x-2)(x+1)$

$= (x-5)(x^2 - 1x - 2)$

17. Prove that $x + a$ is a factor of $(x+a)^5 + (x+c)^5 + (a-c)^5$.

$x + a = 0$

$x = -a$

$f(x) = (x+a)^5 + (x+c)^5 + (a-c)^5$

$f(-a) = \cancel{(-a+a)^5} + (-a+c)^5 + (a-c)^5$

$= (-a+c)^5 + (a-c)^5$

$= (-1(a-c))^5 + (a-c)^5$

$= (-1)^5(a-c)^5 + (a-c)^5$

$= -(a-c)^5 + (a-c)^5$

$= 0$

$\therefore$ by factor theorem

$f(-a) = 0$

$\therefore$ $x+a$ is a factor

3.7 Factoring a Sum or Difference of Cubes

Math Learning Target:
"I can factor fully a Sum **or** Difference of Cubes." $\Delta^3 \pm \square^3$

Sum of Cubes: A^3+B^3

Difference of Cubes: A^3-B^3

Ex.1 Apply the Factor Theorem to factor completely:

a) $y^3 + 8 = P(y)$

$P(-2) = (-2)^3+8$
$= -8+8$
$= 0$
$\therefore (y+2)$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & \downarrow & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0R \end{array}$$

Expected

Now, $y^3+8 = (y+2)(y^2-2y+4)$

b^2-4ac
$=(-2)^2-4(1)(4)$
$=4-16$
$=-12$
$\therefore b^2-4ac<0$
$\therefore$ No Real Roots

b) $8x^3 - 27 = P(x)$

Hint: $P(\frac{3}{2}) = 8(\frac{3}{2})^3 - 27$
$= 8\left(\frac{27}{8}\right)-27$
$= 27-27$
$= 0$

$$\begin{array}{r|rrrr} \frac{3}{2} & 8 & 0 & 0 & -27 \\ & \downarrow & 12 & 18 & 27 \\ \hline & 8 & 12 & 18 & 0R \end{array}$$

$\frac{3}{\cancel{2}}(\cancel{8})^4$
$\frac{3}{\cancel{2}}(\cancel{12})^6$
$\frac{3}{\cancel{2}}(\cancel{18})^9$

$8x^3-27 = (x-\frac{3}{2})(8x^2+12x+18)$
$= (x-\frac{3}{2})2(4x^2+6x+9)$
$= (2x-3)(4x^2+6x+9)$

b^2-4ac
$=(6)^2-4(4)(9)$
$=36-144$
$=-108$
$\therefore b^2-4ac<0$
$\therefore$ No Real Roots

c) $a^3 - b^3 = P(a)$

$P(b) = (b)^3-b^3$
$= 0$
$\therefore a-b$ is a factor

$$\begin{array}{r|rrrr} b & 1 & 0 & 0 & -b^3 \\ & \downarrow & b & b^2 & b^3 \\ \hline & 1 & b & b^2 & 0R \end{array}$$

$\therefore a^3-b^3 = (a-b)(1a^2+ba+b^2)$
$= (a-b)(a^2+ab+b^2)$

b^2-4ac
$=(1)^2-4(1)(1)$
$=1-4$
$=-3$
$\therefore b^2-4ac<0$
$\therefore$ No Real Solutions.

Factor Formula for a Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor Formula for a Sum of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Let's verify the result by expanding:

$$(a - b)(a^2 + ab + b^2)$$

$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$= a^3 - b^3$$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ $\qquad$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Ex.2 Use the appropriate "new" formula to factor completely:

a) $8x^3 - 27$ (from the previous slide)

$$= (2x)^3 - (3)^3 \quad \left\} \begin{array}{l} a = 2x \\ b = 3 \end{array}\right.$$

$$= ((2x) - (3))((2x)^2 + (2x)(3) + (3)^2)$$

$$= (2x - 3)(4x^2 + 6x + 9)$$

b) $27x^3 + 125y^3$

$$= (3x)^3 + (5y)^3$$

$$= (3x + 5y)((3x)^2 - (3x)(5y) + (5y)^2)$$

$$= (3x + 5y)(9x^2 - 15xy + 25y^2)$$

The Factor Theorem can be applied to any expression.
However, it ***may*** be more difficult to use than if one recognizes the expression as a sum/difference of cubes.
Hence, the following algorithm is suggested, from now on, when required to factor:

STOP Is the expression a sum/difference of cubes?
If so, use the appropriate formula.
Otherwise, apply the Factor Theorem directly.

Entertainment: p.182 #2acegi, 3, 4acegi, 5ac, 6

Are you factoring fully?