4.4 Rates of Change in Polynomial Functions

Math Learning Target:

"For any polynomial function,
I can find the average rate of change and rate of change."

Ex. 1: Consider $y = (x+2)^3 + 3$

a) Find the average rate of change on the interval [2, 4]

b) Find the instantaneous rate of change at $x = -1$

c) Find the equation of the tangent line at $x = -1$

a) $\text{aroc} = \dfrac{f(4)-f(2)}{4-2}$

$= \dfrac{(4+2)^3+3-[(2+2)^3+3]}{2}$

$= \dfrac{6^3+3-[4^3+3]}{2}$

$= \dfrac{219-67}{2}$

$= \dfrac{152}{2}$

$= 76$

∴ on average, y increases by 76 units per 1 unit of x, from $x=2$ to $x=4$.

b) $\text{iroc} = m_{\text{tangent}}$

$= \dfrac{f(x+h)-f(x)}{h},\ h \to 0$

$= \dfrac{f(-1+h)-f(-1)}{h},\ h \to 0$

$= \dfrac{(-1+h+2)^3+3-[(-1+2)^3+3]}{h},\ h \to 0$

$= \dfrac{(h+1)^3+3-[(1)^3+3]}{h},\ h \to 0$

$= \dfrac{h^3+3h^2+3h+1+3-[4]}{h},\ h \to 0$

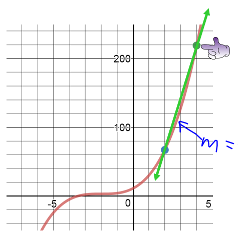
$= \dfrac{h^3+3h^2+3h}{h},\ h \to 0$

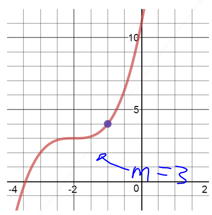
$= \dfrac{\cancel{h}(h^2+3h+3)}{\cancel{h}},\ h \to 0$

$= h^2+3h+3,\ h \to 0$

$= 3$

∴ at $x=-1$, the y is increasing by 3 units per 1 unit of x.





Entertainment:
Worksheet 4.4 #1 & 2

c) eq'n of the tangent at $x=-1$

$y = 3x + b$ at $f(-1) = (-1+2)^3 + 3$

$= 4$

∴ $(-1, 4)$

$4 = 3(-1) + b$

$4 = -3 + b$

$4 + 3 = b$

$b = 7$

∴ $y = 3x + 7$ is the equation of the tangent at $x = -1$.

Pascal's Triangle

row 0   1

1   1 1

2   1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$(x+y)^2$

$= x^2 + 2xy + y^2$

$(x+y)^3 = 1x^3\cancel{y^0} + 3x^2y^1 + 3x^1y^2 + 1\cancel{x^0}y^3$

$(x+y)^5$

$= x^5 y^0 + \underbrace{\frac{1\cdot 5}{1}}_{5} x^4 y^1 + \underbrace{\frac{5\cdot 4}{2}}_{10} x^3 y^2 + \underbrace{\frac{10\cdot 3}{3}}_{10} x^2 y^3 + \underbrace{\frac{10\cdot 2}{4}}_{5} x y^4 + x^0 y^5$

$(x-2)^5$

$= x^5 + 5x^4(-2)^1 + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 + (-2)^5$

$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

Last Day's Entertainment:

Do #7 first...pp.227-228 #7*ef, 3, 8, 9, 12**, 13**, 14, 15
Challenge: #18
* use desmos to confirm your answers;
** the text has answers rounded in the back, but you must state your answers as exact values

p.227 **7.** Solve the following inequalities algebraically. Confirm your answer with a graph.

f) $x^3 - x^2 - 3x + 3 > -x^3 + 2x + 5$

$x^3 - x^2 - 3x + 3 + x^3 - 2x - 5 > 0$

$2x^3 - x^2 - 5x - 2 > 0$

$P(2) = 0$

$$\begin{array}{r|rrrr} 2 & 2 & -1 & -5 & -2 \\ & \downarrow & 4 & 6 & 2 \\ \hline & 2 & 3 & 1 & 0R \end{array}$$

$(x-2)(2x^2+3x+1) > 0$

$(x-2)(2x+1)(x+1) > 0$

$\therefore x = 2, -\frac{1}{2}, -1$

$x < -1$	$-1 < x < -\frac{1}{2}$	$-\frac{1}{2} < x < 2$	$x > 2$
$(-)(-)(-)$	$(-)(-)(+)$	$(-)(+)(+)$	$(+)(+)(+)$
$= -$	$= +$	$= -$	$= +$
$\therefore$ no	$\therefore$ yes	$\therefore$ no.	$\therefore$ yes!

$\therefore (-1, -\frac{1}{2}) \cup (2, \infty)$

p.228 **12.** A rock is tossed from a platform and follows a parabolic path through the air. The height of the rock in metres is given by $h(t) = -5t^2 + 12t + 14$, where t is measured in seconds.

a) How high is the rock off the ground when it is thrown?

b) How long is the rock in the air?

c) For what times is the height of the rock greater than 17 m?

d) How long is the rock above a height of 17 m?

12a) $h(0) = 14$

b) $h(t) = 0$

$\therefore 0 = -5t^2 + 12t + 14$

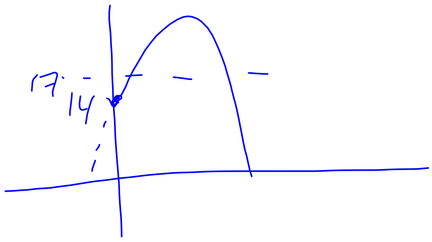
use QRF.

$$t = \frac{-(12) \pm \sqrt{12^2 - 4(-5)(14)}}{2(-5)}$$

$t \doteq -0.85$ or $t \doteq 3.25$

inadmissible

$\therefore$ the rock is in the air for 3.25 seconds.



c) $h(t) > 17$

$-5t^2 + 12t + 14 > 17$

$-5t^2 + 12t + 14 - 17 > 0$

$-5t^2 + 12t - 3 > 0$

$$t = \frac{-12 \pm \sqrt{12^2 - 4(-5)(-3)}}{2(-5)}$$

$$= \frac{-12 \pm \sqrt{84}}{-10}$$

$t = \frac{-12 + \sqrt{84}}{-10}$ or $t = \frac{-12 - \sqrt{84}}{-10}$

$\doteq 0.2834$ $\doteq 2.1165$

$\therefore \{t \in \mathbb{R} \mid 0.28 < t < 2.12\}$

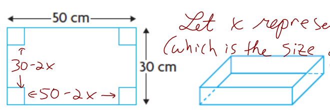
the rock is above 17 m.

d) time above 17m

$\doteq 2.12 - 0.28$

$\doteq 1.84$ seconds

p.228 **13.** An open-topped box can be made from a sheet of aluminium measuring 50 cm by 30 cm by cutting congruent squares from the four corners and folding up the sides. Write a polynomial function to represent the volume of such a box. Determine the range of side lengths that are possible for each square that is cut out and removed that result in a volume greater than 4000 cm^3.



Let x represent the height of the box in cm
(which is the size of the square cut out)

$30-2x \neq 0$

$\therefore x \neq 15$

$V(x) = x(50-2x)(30-2x)$ $\quad * \; 0 \le x < 15$

$x(50-2x)(30-2x) > 4000$

$x(1500 - 100x - 60x + 4x^2) > 4000$

$4x^3 - 160x^2 + 1500x - 4000 > 0$

$f(5) = 0$

$$\begin{array}{r|rrrr} 5 & 4 & -160 & 1500 & -4000 \\ & \downarrow & 20 & -700 & 4000 \\ \hline & 4 & -140 & 800 & 0R \end{array}$$

$\therefore (x-5)(4x^2 - 140x + 800) > 0$

$(x-5)(4(x^2 - 35x + 200)) > 0$

$4(x-5)(x^2 - 35x + 200) > 0$

$x = 5$ or $x = \dfrac{35 \pm \sqrt{425}}{2}$

$= \dfrac{35 \pm \sqrt{25}\sqrt{17}}{2}$

$= \dfrac{35 \pm 5\sqrt{17}}{2}$

$x = \dfrac{35 + 5\sqrt{17}}{2}$ or $x = \dfrac{35 - 5\sqrt{17}}{2}$

$(\doteq 7.19)$ $\quad \doteq 27.81$

$4(x-5)\left(x - \frac{35+5\sqrt{17}}{2}\right)\left(x - \frac{35-5\sqrt{17}}{2}\right) > 0$

$x < 5$	$5 < x < \frac{35-5\sqrt{17}}{2}$	$\frac{35-5\sqrt{17}}{2} < x < 15$
$(-)(-)(-)$	$(+)(-)(-)$	$(+)(+)(-)$
$= -$	$= +$	$= -$
$\therefore$ NO!	$\therefore$ Yes!	$\therefore$ No!

$\therefore$ to have a volume greater than 4000 cm^3

$\left\{x \in \mathbb{R} \mid 5 < x < \frac{35-5\sqrt{17}}{2}\right\}$

if $x = 5$ ← size of square cut

$w = 30 - 2(5)$
$= 30 - 10$
$= 20$

$l = 50 - 2(5)$
$= 50 - 10$
$= 40$

$V = 5(40)(20)$
$= 4000$

$\therefore x > 5$

$l \doteq 39.99$

$w \doteq 19.99$

if $x = \dfrac{35-5\sqrt{17}}{2}$ $(\doteq 7.19)$

$w = 30 - 2\left(\frac{35-5\sqrt{17}}{2}\right)$
$= 30 - (35 - 5\sqrt{17})$
$= -5 + 5\sqrt{17}$
$(\doteq 15.61)$

$l = 50 - 2\left(\frac{35-5\sqrt{17}}{2}\right)$
$= 50 - 35 + 5\sqrt{17}$
$= 15 + 5\sqrt{17}$
$(\doteq 35.61)$

$V = \left(\frac{35-5\sqrt{17}}{2}\right)(15 + 5\sqrt{17})(-5 + 5\sqrt{17})$

$= \left(\frac{35-5\sqrt{17}}{2}\right)(-75 + 75\sqrt{17} - 25\sqrt{17} + 25(17))$

$= \left(\frac{35-5\sqrt{17}}{2}\right)(-75 + 50\sqrt{17} + 425)$

$= \left(\frac{35-5\sqrt{17}}{2}\right)(350 + 50\sqrt{17})$

$= (35 - 5\sqrt{17})(175 + 25\sqrt{17})$

$= 6125 + 875\sqrt{17} - 875\sqrt{17} - 125(17)$

$= 6125 - 2125$

$= 4000$

$\therefore x < \dfrac{35-5\sqrt{17}}{2}$

$x < 7.19$