5.1 Graphs of Reciprocal Functions

Math Learning Target:

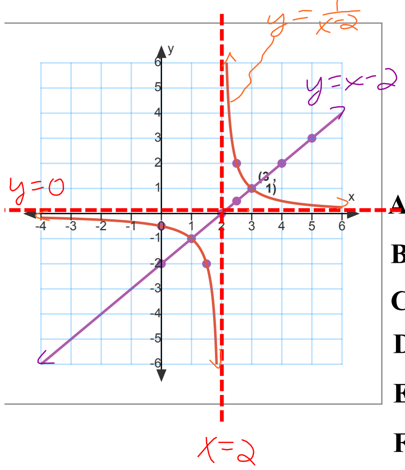
"I know all properties of any linear or quadratic function.
As a result, I immediately know the properties of the reciprocals of these functions.
Finally, I can graph the reciprocal of any linear or quadratic function
using only the associated properties."

For a function $f(x)$ where $f(x) \neq 0$, the **reciprocal function of $f(x)$** is $y = \dfrac{1}{f(x)}$.

Note: the reciprocal function is **not the same** as the **inverse function**

Ex.1: Graph $y = x - 2$ and its reciprocal function $y = \dfrac{1}{x-2}$

a) Graph these functions using transformations of parent functions.



	Property:	$y = x - 2$	$y = \frac{1}{x-2}$
	Domain	$(-\infty, \infty)$	$(-\infty, 2) \cup (2, \infty)$
	Range	$(-\infty, \infty)$	$(-\infty, 0) \cup (0, \infty)$
A.	Zero(s)	$x = 2$	none ∴ V.A. $x = 2$
B.	y-intercept	$y = -2$	$y = -\frac{1}{2}$
C.	Interval(s) where positive	$(2, \infty)$	$(2, \infty)$
D.	Interval(s) where negative	$(-\infty, 2)$	$(-\infty, 2)$
E.	Interval(s) of increase	$(-\infty, \infty)$	none
F.	Interval(s) of decrease	none	$(-\infty, 2) \cup (2, \infty)$
G.	Intersection Points	$(1, -1), (3, 1)$	

A linear function versus its reciprocal (by properties examined above)

A. An x-value that gives $y = 0$ for the line, now gives a value of zero in the denominator of its reciprocal. Division by zero is undefined, creating a vertical asymptote at that point.

B. For every y-value calculated for the line, it can be placed in the denominator of its reciprocal. If $y = a$ is an intercept for the line, it is $y = \dfrac{1}{a}$ for its reciprocal.

C. If $y = a$ is positive, then $y = \dfrac{1}{a}$ is also positive.

D. If $y = a$ is negative, then $y = \dfrac{1}{a}$ is also negative.

E. If numbers for y in one function increase, then the corresponding numbers for $\dfrac{1}{y}$ decrease.

F. If y-values decrease, then the values for $\dfrac{1}{y}$ increase.

G. They must share points when $y = 1$ or $y = -1$, *assuming they belong to the range of the function*.
For example, At $x = 3$

the line
$y = x - 2$
$= 3 - 2$
$= 1$

its reciprocal
$$y = \frac{1}{x-2} = \frac{1}{3-2} = \frac{1}{1} = 1$$

they share (3, 1)

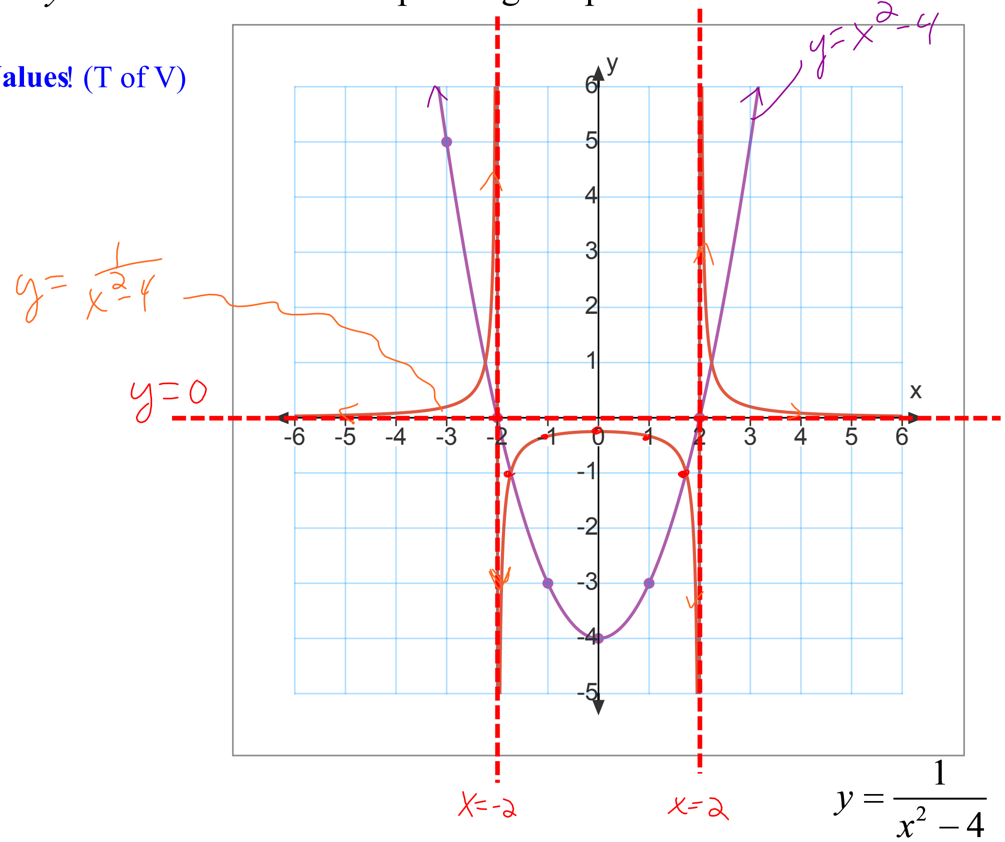
Finally, ALL reciprocal functions always have a horizontal asymptote $y = 0$.

All of the relationships between properties for a line versus its reciprocal, also apply for a quadratic versus its reciprocal.

From now on, all reciprocal functions MUST be graphed according to the relationships between the properties (A to H) of the associated functions.

Ex. 2: Graph $y = x^2 - 4$ and its corresponding reciprocal function.

No Table of Values! (T of V)



$$y = \frac{1}{x^2 - 4}$$

One more property, and the relationship between a quadratic and its reciprocal:

	Property:	$y = x^2 - 4$	$y = \frac{1}{x^2 - 4}$
H.	Local Extrema	$(0,-4)$	$(0,-\frac{1}{4})$

H. A local maximum for one function corresponds to a local minimum for its reciprocal, and vice versa, for a given x.

Before you begin today's entertainment:

1. All graphs from now on cannot be made from a table of values. Use properties A through H;
2. Whenever the text says "sketch", ignore this, create a "graph" instead.

Complete pp. 254-257 #1, 6, 7, 8cf, 11, 12**. Challenge yourself! #15

** there is an incorrect final answer for 12e). It should be: $D: \{t \in R \mid 0 \le t \le 10000\}$

$R: \{b \in W \mid 0 \le b \le 10000\}$