5.3 Graphs of the form: $f(x)=\dfrac{ax+b}{cx+d}$

Math Learning Target:

"I can easily determine horizontal asymptotes when the function is of the above form. Moreover, I can graph functions of the above form."

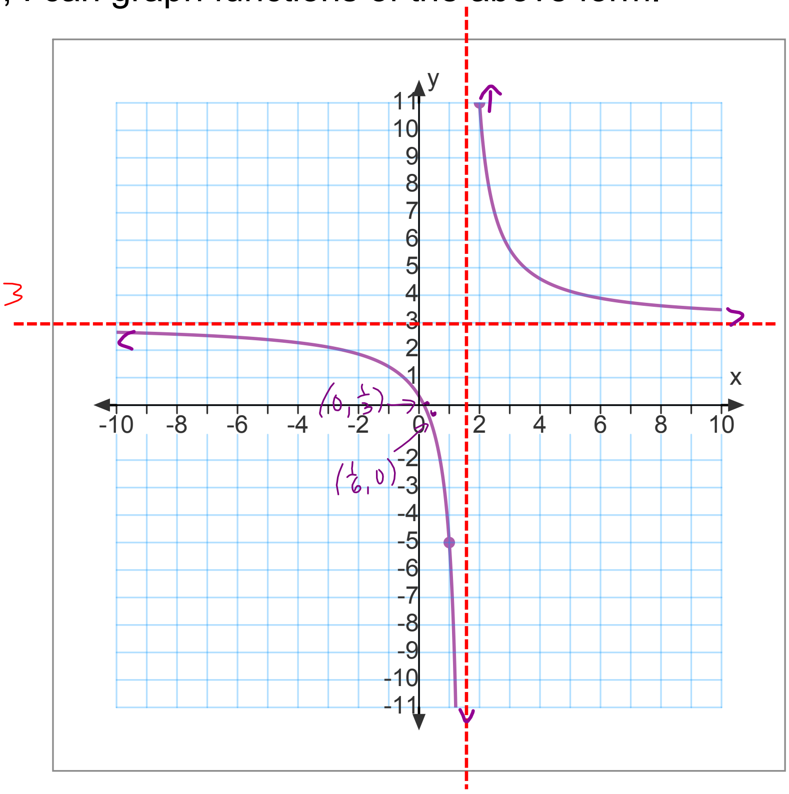
Ex.1: Graph $y=\dfrac{6x-1}{2x-3}$

H.A. ∵ degree of numerator = degree of denominator

$\therefore y=\frac{6}{2}\left(\frac{x}{x}\right)$

$y=3$ is the H.A.

$y=3$



Restriction: $2x-3\neq 0$

$x\neq \frac{3}{2}$

$f\left(\frac{3}{2}\right)=\dfrac{6\left(\frac{3}{2}\right)-1}{2\left(\frac{3}{2}\right)-3}$

$=\frac{8}{0}$

$\therefore$ No hole

$\therefore$ V.A.: $x=\frac{3}{2}$

$y=\dfrac{6x-1}{2x-3}$ does not simplify

$x=\frac{3}{2}$

Near

y-int, let $x=0$

$f(0)=\dfrac{6(0)-1}{2(0)-3}$

$=\frac{1}{3}$

x-int, let $y=0$

$0=\dfrac{6x-1}{2x-3}$

$0=6x-1$

$x=\frac{1}{6}$

$f(1.49)$

$=-397$

$f(1.51)$

$=\dfrac{6(1.51)-1}{2(1.51)-3}$

$=\frac{8.06}{0.02}$

$=403$

$f(2)=\dfrac{6(2)-1}{2(2)-3}$

$=\frac{11}{1}$

$=11$

$f(-1)=\dfrac{6(1)-1}{2(1)-3}$

$=\frac{5}{-1}$

$=-5$

$f(-100)$

$=\frac{-601}{-203}$

$\doteq 2.96$

(below H.A.)

$f(100)=\frac{599}{197}$

$\doteq 3.04$

(above H.A.)

Entertainment: p. 272 #1, 5ad, 6, 8*, 9, 10**

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Answers that need to be corrected in the text:

8* $f(x)$ has a VA at $x=1$; $g(x)$ has a HA at $y=0.5$.
Also, $f(x)$ has a HA at $y=3$; $g(x)$ has a VA at $x=-1.5$

10** The concentration increases over the 24 h period and approaches approx. 1.85 mg/L

14***a) $f(x)$ and $m(x)$ b) $g(x)$

Find the equation of the H.A. for $f(x)=\dfrac{ax+b}{cx+d}$

We know a horizontal asymptote exists for the above function,
since the degree of the numerator is the same as the degree of the denominator.

Now any HA exists only as $x \longrightarrow \infty$, and/or $x \longrightarrow -\infty$.

$$y=\frac{ax+b}{cx+d} \qquad \left.\begin{matrix}\div x\\ \div x\end{matrix}\right\} \begin{matrix}\text{allowed, since } x \longrightarrow \pm\infty\\ \text{i.e. } x \neq 0\end{matrix}$$

$$=\frac{\frac{ax}{x}+\frac{b}{x}}{\frac{cx}{x}+\frac{d}{x}}$$

$$=\frac{a+\frac{b}{x}}{c+\frac{d}{x}}$$

As $x \to \pm\infty$, $\frac{b}{x} \to 0$

$$y=\frac{a+0}{c+0}$$

$$=\frac{a}{c} \quad \therefore \text{the H.A.}$$