

Then complete the **Warm-up** question: Solve

$$\frac{1}{3}x - \frac{1}{5} = \frac{x}{6}$$

$$\text{LCD} = 30$$

Better Way

$$\overset{10}{\cancel{30}} \left(\frac{1}{\cancel{3}_1} x \right) - \overset{6}{\cancel{30}} \left(\frac{1}{\cancel{5}_1} \right) = \overset{5}{\cancel{30}} \left(\frac{x}{\cancel{6}_1} \right)$$

$$10x - 6 = 5x$$

$$10x - 5x = 6$$

$$5x = 6$$

$$x = \frac{6}{5}$$

Another Way

$$\left(\frac{1}{3} \right) \left(\frac{10}{10} \right) x - \left(\frac{1}{5} \right) \left(\frac{6}{6} \right) = \frac{x}{6} \left(\frac{5}{5} \right)$$

$$\frac{10x}{30} - \frac{6}{30} = \frac{5x}{30}$$

$$\frac{10x}{30} - \frac{5x}{30} = \frac{6}{30}$$

$$\frac{5x}{30} = \frac{6}{30}$$

$$150x = 180$$

$$x = \frac{180}{150}$$

$$= \frac{18}{15}$$

$$= \frac{6}{5}$$

Last Day's Work:: p. 272 #1, 5ad, 6, 8*, 9, 10**

Enrich Yourself!... p. 274 #12, 13, 14***

Answers that need to be corrected in the text:

8* $f(x)$ has a VA at $x=1$; $g(x)$ has a HA at $y=0.5$.

Also, $f(x)$ has a HA at $y=3$; $g(x)$ has a VA at $x=-1.5$

10** The concentration increases over the 24 h period and approaches approx. 1.85 mg/L

14*** a) $f(x)$ and $m(x)$ b) $g(x)$

- p. 274 8. Without using a graphing calculator, compare the graphs of the rational functions $f(x) = \frac{3x+4}{x-1}$ and $g(x) = \frac{x-1}{2x+3}$.

$$f(x) = \frac{3x+4}{x-1}$$

$$g(x) = \frac{x-1}{2x+3}$$

$$\text{VA: } x = 1$$

$$\text{VA: } x = -\frac{3}{2}$$

$$\text{HA: } y = 3$$

$$\text{HA: } y = \frac{1}{2}$$

$$\text{y-int: } f(0) = -4$$

$$\text{y-int: } g(0) = -\frac{1}{3}$$

$$\text{x-int: let } f(x) = 0$$

$$\text{x-int, let } g(x) = 0$$

$$\therefore x = -\frac{4}{3}$$

$$\therefore x = 1$$

p. 274

9. The function $I(t) = \frac{15t + 25}{t}$ gives the value of an investment, in thousands of dollars, over t years.

- What is the value of the investment after 2 years?
- What is the value of the investment after 1 year?
- What is the value of the investment after 6 months?
- There is an asymptote on the graph of the function at $t = 0$. Does this make sense? Explain why or why not. \rightarrow you can't invest for 0 years.
- Choose a very small value of t (a value near zero). Calculate the value of the investment at this time. Do you think that the function is accurate at this time? Why or why not?
- As time passes, what will the value of the investment approach?

$$9a) I(2) = \frac{15(2) + 25}{2} \quad b) I(1) = \frac{15(1) + 25}{1} \quad c) 6 \text{ months} = \frac{6}{12} \text{ year}$$

$$= \frac{55}{2} \quad = 40 \quad = 0.5$$

$$= 27.5 \quad \therefore \$40,000$$

$$\therefore \$27,500$$

$$I(0.5) = \frac{15(0.5) + 25}{0.5}$$

$$= \frac{32.5}{0.5}$$

$$= 65$$

$$\therefore \$65,000$$

f) we need H.A.

$$y = \frac{15}{1}$$

$$= 15$$

$$\therefore \$15,000$$

5.4 Solving Rational Equations

Math Learning Target:



"I can state the restrictions on the variable in any rational equation. Then, I can solve the equation both algebraically and graphically. Finally, I can construct (and solve) a rational equation that arises from a real application."

Ex. 1: Solve $\{x \in \mathbb{R}\}$. $\frac{1}{3x} - \frac{1}{2} = \frac{5}{6x}$

$$\cancel{6x} \left(\frac{1}{\cancel{3x}} \right) - \frac{\cancel{3x}}{\cancel{6x}} \left(\frac{1}{2} \right) = \frac{5}{\cancel{6x}} \left(\frac{1}{\cancel{6x}} \right)$$

$$2 - 3x = 5$$

$$-3x = 5 - 2$$

$$-3x = 3$$

$$\therefore x = -1$$

Restriction:

$$x \neq 0$$

$$\text{LCD} = 6x$$

$$\frac{1}{3x} \left(\frac{2}{2} \right) - \frac{1}{2} \left(\frac{3x}{3x} \right) = \frac{5}{6x} \left(\frac{1}{1} \right)$$

$$\frac{2}{6x} - \frac{3x}{6x} = \frac{5}{6x}$$

$$\frac{2 - 3x}{6x} = \frac{5}{6x}$$

$$\frac{-3}{6x} = \frac{3x}{6x}$$

$$\therefore -3 = 3x$$

$$\therefore x = -1$$

Warm-up

$$\frac{1}{3}x - \frac{1}{5} = \frac{x}{6}$$

Ex. 2

a) Determine a function whose zeros are the solutions to:

$\hookrightarrow f(x)$

$$\frac{5}{4} = \frac{1}{x} - \frac{1}{x-5}$$

Restrictions:

$$x \neq 0, 5$$

$$0 = \frac{1}{x} - \frac{1}{x-5} - \frac{5}{4}$$

$$f(x) = \frac{1}{x} - \frac{1}{x-5} - \frac{5}{4}$$

$$\text{LCD} = 4x(x-5)$$

$$= \frac{1}{x} \left(\frac{4(x-5)}{4(x-5)} \right) - \frac{1}{(x-5)} \left(\frac{4x}{4x} \right) - \frac{5}{4} \left(\frac{x(x-5)}{x(x-5)} \right)$$

$$= \frac{1(4(x-5))}{4x(x-5)} - \frac{1(4x)}{4x(x-5)} - \frac{5(x(x-5))}{4x(x-5)}$$

$$= \frac{4x - 20 - 4x - 5x^2 + 25x}{4x(x-5)}$$

$$= \frac{-5x^2 + 25x - 20}{4x(x-5)}$$

b) Solve for the zeros algebraically $\{x \in \mathbb{R}\}$. Check your solution

$$\frac{5}{4} = \frac{1}{x} - \frac{1}{x-5} \quad \text{Rest: } x \neq 0, 5$$

$$\text{LCD} = 4x(x-5)$$

$$\cancel{4x(x-5)} \left(\frac{5}{4} \right) = 4x(x-5) \left(\frac{1}{x} \right) - 4x(x-5) \left(\frac{1}{x-5} \right)$$

$$5x(x-5) = 4(x-5) - 4x(1)$$

$$5x^2 - 25x = 4x - 20 - 4x$$

$$5x^2 - 25x = -20$$

$$5x^2 - 25x + 20 = 0$$

$$5(x^2 - 5x + 4) = 0$$

$$5(x-4)(x-1) = 0$$

$$\therefore x = 4 \text{ or } x = 1$$

check $x = 4$

$$LS = \frac{5}{4} \quad RS = \frac{1}{x} - \frac{1}{x-5}$$

$$= \frac{1}{4} - \frac{1}{4-5}$$

$$= \frac{1}{4} - \frac{1}{-1}$$

$$= \frac{1}{4} + 1$$

$$\therefore LS = RS = \frac{1}{4} + \frac{4}{4}$$

$$\therefore x = 4 = \frac{5}{4}$$

is a solution.

check $x = 1$

$$LS = \frac{5}{4}$$

$$RS = \frac{1}{x} - \frac{1}{x-5}$$

$$= \frac{1}{1} - \frac{1}{1-5}$$

$$= 1 - \frac{1}{-4}$$

$$= 1 + \frac{1}{4}$$

$$= \frac{5}{4}$$

$$\therefore LS = RS$$

$\therefore x = 1$ is a zero.

Entertainment:

pp. 285-287 #3b, 4b (do not "verify"), 5c, 6abc, *9, 11 (see Example 4, text), **12

Challenge: #16 (use [desmos](#)).

Answer for #16a) should be: at 0.417 sec and 1.705 sec.

Legend:

** final answers must be stated as simplified exact values (not rounded!)

Extra: Solving a quadratic **EQUATION** by completing the square.

From Ex.2b

$$x^2 - 5x + 4 = 0$$

$$x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 4 = 0$$

$$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{16}{4} = 0$$

$$\left(x - \frac{5}{2}\right)^2 - \frac{9}{4} = 0$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{9}{4}$$

$$x - \frac{5}{2} = \pm \sqrt{\frac{9}{4}}$$

$$x - \frac{5}{2} = \pm \frac{3}{2}$$

$$x = \frac{5}{2} \pm \frac{3}{2}$$

$$x = \frac{5}{2} + \frac{3}{2} \quad \text{or} \quad x = \frac{5}{2} - \frac{3}{2}$$

$$= \frac{8}{2} \quad \quad \quad = \frac{2}{2}$$

$$= 4 \quad \quad \quad = 1$$