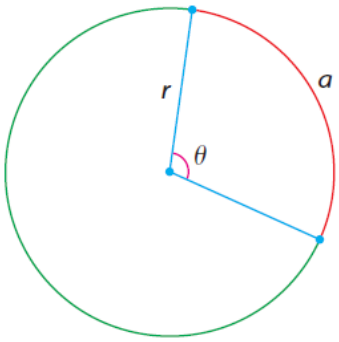
Unit 6: Trigonometric Functions

6.1 Radian Measure

Math Learning Target:

"I understand how to calculate a radian measure.
Also, I can convert an angle in degrees to the same angle expressed in radians.
Finally, I can solve problems involving angular velocity."



The size of the angle is expressed in terms of the arc length, a, that subtends the angle θ at the centre of a circle with radius, r.

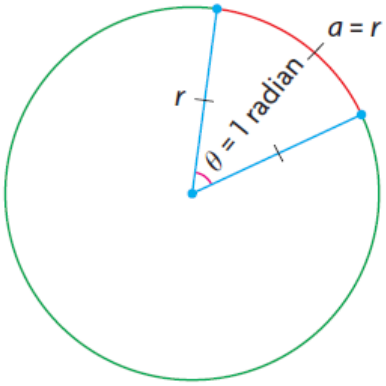
In this situation, a is proportional to both r and θ.

Hence,

$$\theta = \frac{a}{r}$$

This unit of measure, θ is the **radian**.

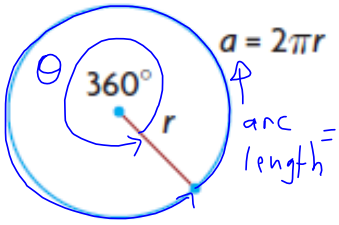
1 radian is defined when $a = r$.



As you see in the picture,

it appears as though 1 radian should be a little less than 60°, since the sector of the circle formed *resembles* an equilateral triangle (but with one side that is curved).

How does one convert between radians and degrees?



$C = 2\pi r$

1 revolution = C

$\theta = \frac{a}{r}$

$\theta = \frac{2\pi r}{r}$

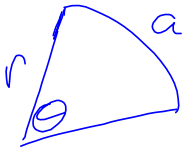
$\theta = 2\pi$

$360^\circ = 2\pi$ radians

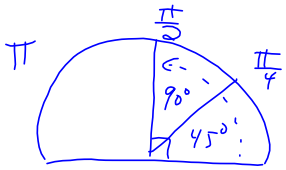
$180^\circ = \pi$ radians

$\frac{180^\circ}{\pi} = 1$ radian or $1^\circ = \frac{\pi}{180^\circ}$

$\therefore$ 1 radian $\doteq 57.3^\circ$



$\theta = \frac{a}{r}$



Ex.1: Convert to radians.

a) 40°

$\frac{40^\circ}{360^\circ} = \frac{x}{2\pi}$

$2\pi\left(\frac{40^\circ}{360^\circ}\right) = x$

$\frac{2\pi}{9} = x$ radians

$\pi = 180^\circ$

$40^\circ \times \frac{\pi}{180^\circ}$

$= \frac{2\pi}{9}$

b) 315°

$= 315^\circ \times \frac{\pi}{180^\circ}$

$= \frac{7\pi}{4}$ radians

Ex. 2: Convert from radians to degrees.

a) $\frac{2\pi}{3}$

$= \frac{2\pi}{3} \times \frac{180^\circ}{\pi}$

$= 120^\circ$

b) $-\frac{3\pi}{4}$

$= -\frac{3\pi}{4} \times \frac{180^\circ}{\pi}$

$= -135^\circ$

$= \frac{-3(180^\circ)}{4}$

$= -135^\circ$

Ex. 3: A wind turbine with three blades rotates five times per minute.

a) What is the angular velocity in radians per second?

$\text{angular velocity} = \frac{\Delta\theta}{\Delta t}$

$\text{velocity} = \frac{\Delta d}{\Delta t}$

1 revolution = 2π radians

5 rev = 10π radians

$\therefore \frac{\Delta\theta}{\Delta t} = \frac{10\pi \text{ radians}}{1 \text{ minute}}$

$= \frac{10\pi \text{ radians}}{60 \text{ sec}}$

$= \frac{\pi}{6}$ radians/sec

$\therefore$ the angular velocity is $\frac{\pi}{6}$ radians/sec.

b) The radius of the turbine is 15 *m*.

How far does the tip of the blade travel after 3 minutes?

1 revolution = circumference

$C = 2\pi r$

$= 2\pi(15)$

$= 30\pi$ m

In 1 min, $d = 30\pi \times 5$

$= 150\pi$ m

$\therefore$ In 3 min, $d = 150\pi \times 3$

$= 450\pi$ m

$\therefore$ the tip of the blade travels 450π m in 3 min.

Entertainment:

pp. 320-322 #1aceg, 2aceg, 3bc, 4bc, 5, 7ab, 8ab, 9ac, 11, 12, 13.

Challenge Yourself! #10, 16* the answer for 16 should be about 86.81 radians per second.