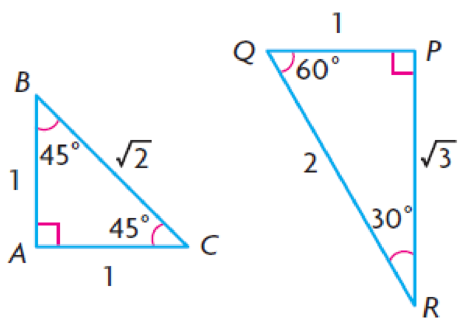
6.2 Radian Measure and Angles on the Cartesian Plane

Math Learning Target:

"I can evaluate any trigonometric ratio for any angle expressed in radians on the Cartesian Plane. Also, if I am given any angle in radians on the Cartesian Plane, I can find its corresponding trigonometric ratios in simplest form. Finally, without a calculator, I can meet the previous two targets, if the angles in question are made from the Special Triangles, or are multiples of $\frac{\pi}{2}$ radians."

Recall: "Special Triangles"



Memorize this Chart!

θ	30° $\frac{\pi}{6}$	45° $\frac{\pi}{4}$	60° $\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Pull

Recall: Primary Trig Ratios: "soh-cah-toa"

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

Recall: Reciprocal Trig Ratios

$\csc\theta = \frac{hyp}{opp}$ $\sec\theta = \frac{hyp}{adj}$ $\cot\theta = \frac{adj}{opp}$

cosecant   secant   cotangent

"Rule of co"

Recall: "syr-cxr-tyx"

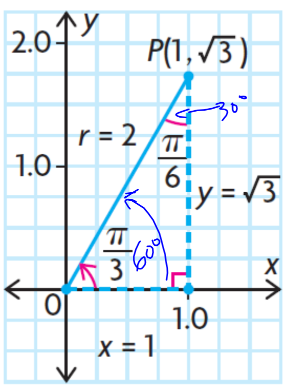
i.e.

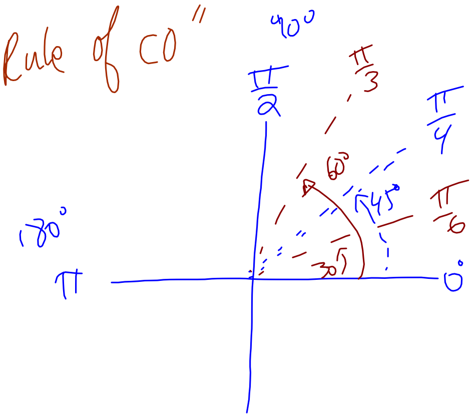
$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

where $x^2 + y^2 = r^2$ and $r > 0$.



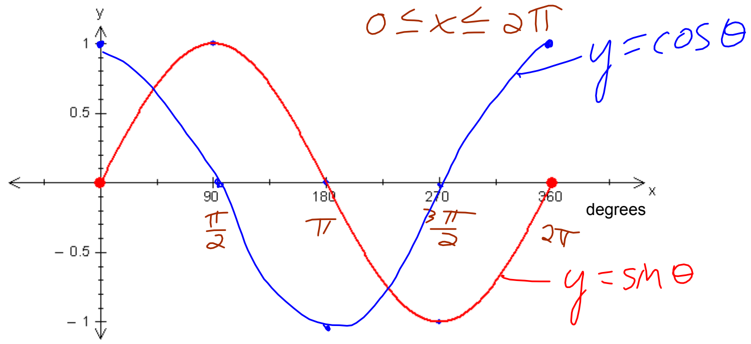


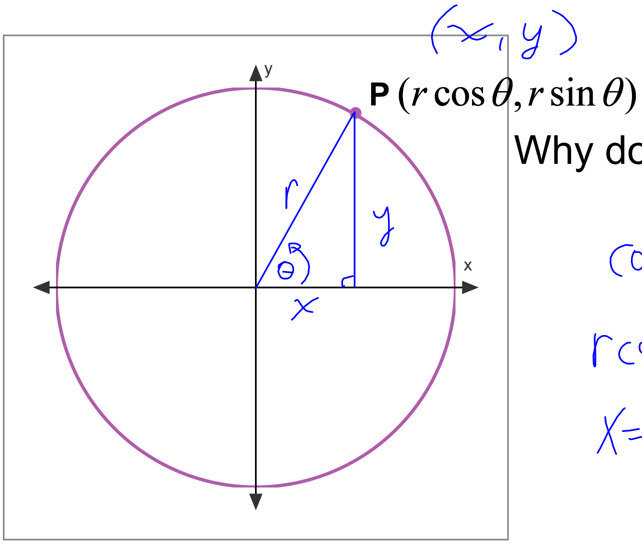
Without using a calculator, you must know how to (quickly) determine all trig ratios (in exact form) for each special triangle.

<u>Recall:</u> Graphs of Sine and Cosine

$0^\circ \leq x \leq 360^\circ$

$0 \leq x \leq 2\pi$





Why does point P have these coordinates?

$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$

$r\cos\theta = x$ $y = r\sin\theta$

$x = r\cos\theta$

Using a circle of radius r, $\Leftarrow r = 1$

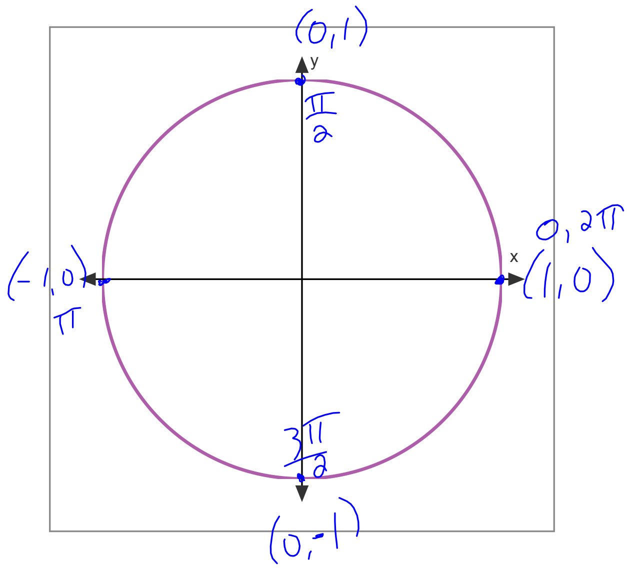
prove $\cos\left(\frac{\pi}{2}\right) = 0$ and $\sin\left(\frac{\pi}{2}\right) = 1$.

$\therefore x = 0,\ y = 1,\ r = 1$

$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$

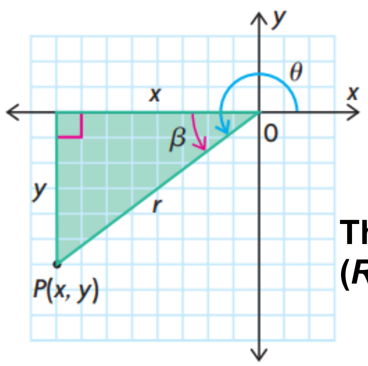
$\cos\left(\frac{\pi}{2}\right) = \frac{0}{1}$ $\sin\left(\frac{\pi}{2}\right) = \frac{1}{1}$

$\cos\left(\frac{\pi}{2}\right) = 0$ $\sin\frac{\pi}{2} = 1$



Without using a calculator, you must know how to (quickly) determine all trig ratios for angles in multiples of $\frac{\pi}{2}$.

The trigonometric ratios for any principal angle, θ, in standard position can be determined by finding the related acute angle, β, using coordinates of any point that lies on the terminal arm of the angle.



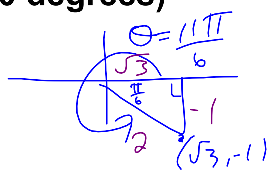
From the Pythagorean theorem, $r^2 = x^2 + y^2$, if $r > 0$.

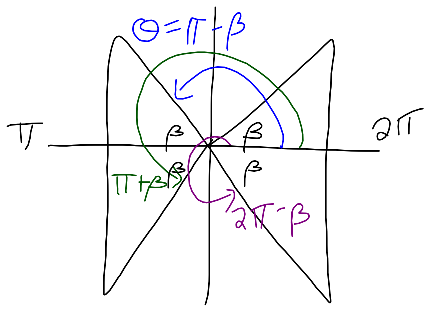
$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$

The angle β is termed the <u>related acute angle</u> (or raa).
(*Recall:* <u>acute</u> refers to an angle greater than 0 degrees but less than 90 degrees)

Ex.1: Determine the exact value: $\sec\left(\frac{11\pi}{6}\right)$





Method 1

$\beta = 2\pi - \frac{11\pi}{6}$

$= \frac{12\pi}{6} - \frac{11\pi}{6}$

$= \frac{\pi}{6}$ (raa)

$\sec\theta = \frac{r}{x}$

$\sec\frac{11\pi}{6} = \frac{2}{\sqrt{3}}$

Method 2

$\sec\left(\frac{11\pi}{6}\right)$

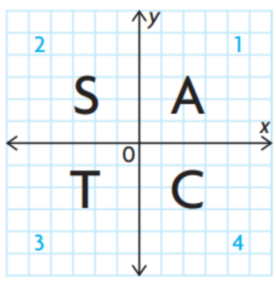
$= +\sec\left(\frac{\pi}{6}\right)$

$= \frac{1}{\cos\left(\frac{\pi}{6}\right)}$

$= \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

cos is +ve
∴ sec is also +ve



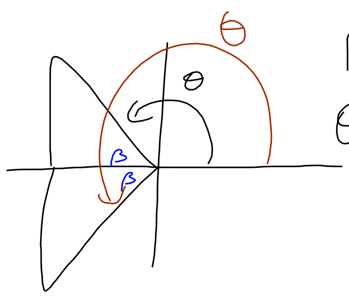
When determining an angle using your calculator, it <u>always</u> gives you *one* angle created with the terminal arm, relative to the positive side of the x-axis.

Ex.2: If $\cos\theta = -\frac{5}{11}$, where $0 \le \theta \le 2\pi$, evaluate θ to the nearest hundredth.

use $\cos\beta = \frac{5}{11}$

$\beta = \cos^{-1}\left(\frac{5}{11}\right)$

$\doteq 1.098$



In QII

$\theta = \pi - \beta$

$\doteq \pi - 1.098$

$\doteq 2.042$

$\doteq 2.04$

In QIII

$\theta = \pi + \beta$

$\doteq \pi + 1.098$

$\doteq 4.240$

$\doteq 4.24$

Read p.329

<u>Entertainment</u>: pp.330-332 #2ab, 3, 5acdf, 6cdef, 7ad, 11, 13, 15, 16, 19