6.4 Transformations of Trigonometric Functions

Math Learning Target: $f(x) = a\cos[k(x-d)] + c$

"I can identify and graph any transformations of the trigonometric functions studied."

Ex.1: Graph for one period: $f(x) = -2\cos\left(\dfrac{1}{2}x - \dfrac{\pi}{2}\right) + 1$ RST

reflection in the x-axis $= -2\cos\left[\dfrac{1}{2}(x - \pi)\right] + 1$

V.S. by a factor of 2

h.t. π units to the right

$\hookrightarrow$ phase shift π units to the right $period = \dfrac{orig}{|k|}$

h.S by a factor of 2.

$= \dfrac{2\pi}{\frac{1}{2}}$

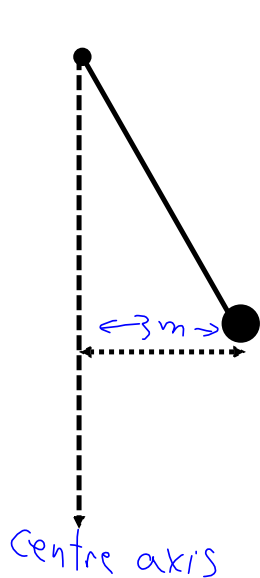U.t up 1 unit

$= 2\pi \div \frac{1}{2}$

$= 2\pi \times 2$

$= 4\pi$
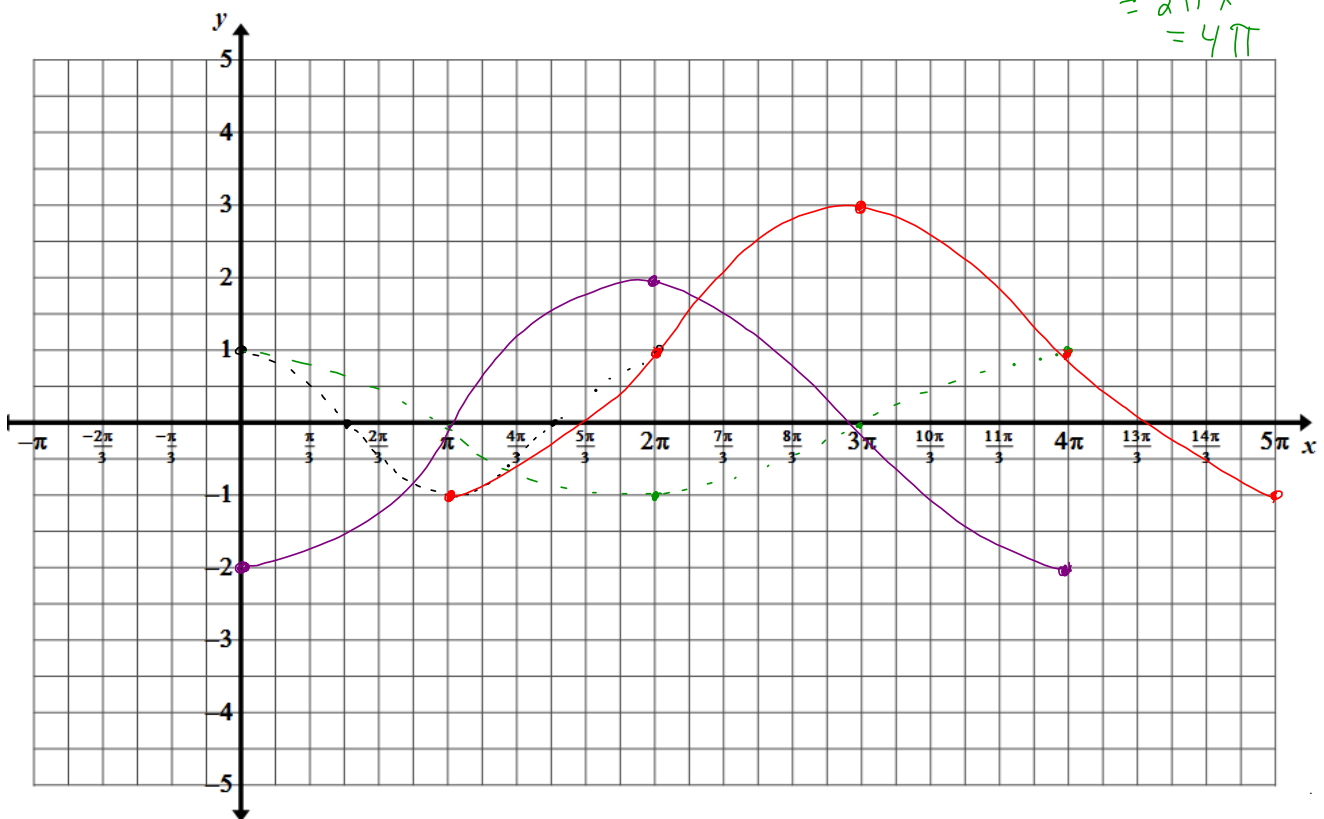
Ex. 2: A large pendulum swings back and forth with a maximum of a 3 m horizontal distance from its centre, and completes one cycle every 4 seconds.

a) Graph this motion for two cycles, beginning at the end of its swing.

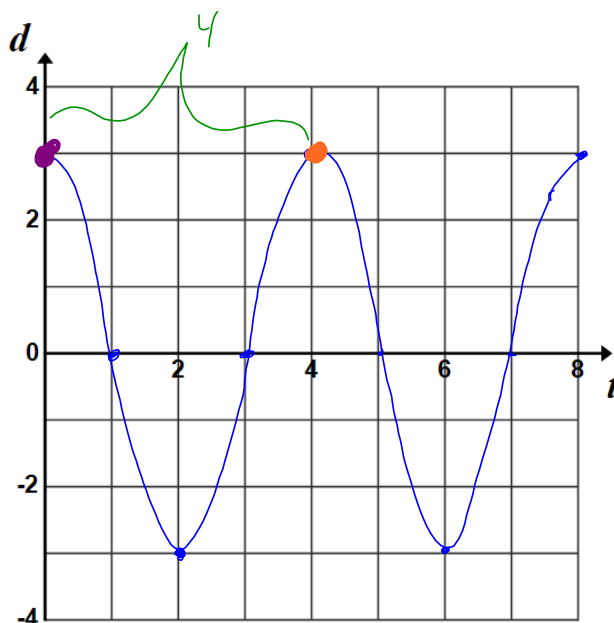b) Write one equation to model the horizontal distance from its centre, d, over time, t.

Centre axis

$f(x) = a \cos[k(x-d)] + c$

$d = 3\cos\left(\frac{\pi}{2}t - 0\right) + 0$

$\quad = 3\cos\left(\frac{\pi}{2}t\right)$

or $d = 3\cos\left(\frac{\pi}{2}(t-4)\right) + 0$

$|a| = \dfrac{max - min}{2}$

$\quad = \dfrac{3 - (-3)}{2}$

$\quad = \dfrac{6}{2}$

$\quad = 3$

$|k| = \dfrac{orig}{period}$

$\quad = \dfrac{2\pi}{4}$

$\quad = \dfrac{\pi}{2}$

$d = 0$

$c = \dfrac{max + min}{2}$

$\quad = \dfrac{3 + (-3)}{2}$

$\quad = \dfrac{0}{2}$

$\quad = 0$

Entertainment: pp.343-346 #**1ad, 4bc, 5ac, 6c, 7bc, 8c*d*** <u>graph</u> (do not sketch), **9, 10*** <u>graph</u> (do not sketch), **11, 12, 14a**

6.3 Entertainment: p. 336 #2c, 3, 5

6.2
Entertainment: pp.330-332 #2ab, 3, 5acdf, 6cdef, 7ad, 11, 13, 15, 16, 19
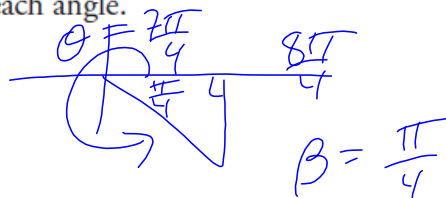
cd

3. Determine the primary trigonometric ratios for each angle.

a) $-\dfrac{\pi}{2}$

b) $-\pi$

c) $\dfrac{7\pi}{4}$

d) $-\dfrac{\pi}{6}$

$\theta = \dfrac{7\pi}{4}$ $\dfrac{8\pi}{4}$

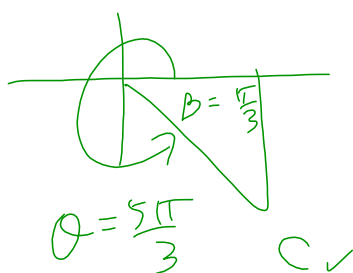$\dfrac{\pi}{4}$

$\beta = \dfrac{\pi}{4}$

c) $\dfrac{7\pi}{4}$

$\sin \dfrac{7\pi}{4} = -\sin \dfrac{\pi}{4}$ $\cos \dfrac{7\pi}{4} = +\cos \dfrac{\pi}{4}$ $\tan \dfrac{7\pi}{4} = -\tan \dfrac{\pi}{4}$

$\qquad = -\dfrac{\sqrt{2}}{2}$ $\qquad = \dfrac{\sqrt{2}}{2}$ $\qquad = -1$

d) $-\dfrac{\pi}{6}$ $\rightarrow \sin\left(-\dfrac{\pi}{6}\right)$ $\cos\left(-\dfrac{\pi}{6}\right)$ $\tan\left(-\dfrac{\pi}{6}\right)$

$\beta = -\dfrac{\pi}{6}$ $= -\sin\left(\dfrac{\pi}{6}\right)$ $= +\cos\left(\dfrac{\pi}{6}\right)$ $= -\tan\left(\dfrac{\pi}{6}\right)$

$\qquad = -\dfrac{1}{2}$ $\qquad = \dfrac{\sqrt{3}}{2}$ $\qquad = -\dfrac{\sqrt{3}}{3}$

5. Determine the exact value of each trigonometric ratio.

f) $\sec \dfrac{5\pi}{3}$

$\beta = \dfrac{\pi}{3}$

$\theta = \dfrac{5\pi}{3}$ C ✓

$\beta = 2\pi - \theta$

$\quad = \dfrac{6\pi}{3} - \dfrac{5\pi}{3}$

$\quad = \dfrac{\pi}{3}$

$= +\sec\left(\dfrac{\pi}{3}\right)$

$= \dfrac{1}{\cos\left(\dfrac{\pi}{3}\right)}$

$= \dfrac{1}{\dfrac{1}{2}}$

$= 2$

11. A clock is showing the time as exactly 3:00 p.m. and 25 s. Because a
    T full minute has not passed since 3:00, the hour hand is pointing
    directly at the 3 and the minute hand is pointing directly at the 12. If
    the tip of the second hand is directly below the tip of the hour hand,
    and if the length of the second hand is 9 cm, what is the length of the
    hour hand?