

6.1 Entertainment:

pp. 320-322 #1aceg, 2aceg, 3bc, 4bc, 5, 7ab, 8ab, 9ac, 11, 12, 13.

Challenge Yourself! #10, 16* the answer for 16 should be about 86.81 radians per second.

6.2 Entertainment: pp.330-332 #2ab, 3, 5acdf, 6cdef, 7ad, 11, 13, 15, 16, 19

6.3 Entertainment: p. 336 #2c, 3, 5

6.4 Entertainment: pp.343-346 #1ad, 4bc, 5ac, 6c, 7bc, 8c*d* graph (do not sketch),
9, 10* graph (do not sketch), 11, 12, 14a

6.1 Entertainment:

pp. 320-322 #1aceg, 2aceg, 3bc, 4bc, 5, 7ab, 8ab, 9ac, 11, 12, 13.Challenge Yourself! #10, 16* the answer for 16 should be about 86.81 radians per second.

- p.322 11. A wind turbine has three blades, each measuring 3 m from centre to tip. At a particular time, the turbine is rotating four times a minute.
- Determine the angular velocity of the turbine in radians/second.
 - How far has the tip of a blade travelled after 5 min?

$$\begin{aligned}
 & 11. \quad r = 3\text{m} \quad 4\text{ rev/min.} \\
 & \text{a) } \text{ang. vel.} = \frac{\theta}{t} \quad \left. \begin{array}{l} \theta = 2\pi \times 4 \\ = 8\pi \text{ rad.} \end{array} \right\} \\
 & \quad = \frac{8\pi \text{ rad}}{1\text{min}} \\
 & \quad = \frac{8\pi}{1\text{min}} \times \frac{1\text{min}}{60\text{sec}} \\
 & \quad = \frac{4\pi}{30} \text{ rad/sec} \\
 & \quad = \frac{2\pi}{15} \text{ rad/sec} \\
 & \quad \approx 0.418 \text{ rad/sec}
 \end{aligned}$$

$$\begin{aligned}
 & \text{b) } 1 \text{ rev.} = 2\pi r \\
 & \quad = 2\pi(3) \\
 & \quad = 6\pi \\
 & 4\text{ rev/min} = 24\pi / \text{min} \\
 & \therefore \text{ after } 5\text{min} = 120\pi \text{ m} \\
 & \quad \approx 376.99 \\
 & \quad \approx 377 \text{ m}
 \end{aligned}$$

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pp. 320-322 #1aceg, 2aceg, 3bc, 4bc, 5, 7ab, 8ab, 9ac, 11, 12, 13.

Challenge Yourself! #10, 16* the answer for 16 should be about 86.81 radians per second.

- p.322 12. A wheel is rotating at an angular velocity of 1.2π radians/s, while a point on the circumference of the wheel travels 9.6π m in 10 s.
- How many revolutions does the wheel make in 1 min?
 - What is the radius of the wheel?

(w)

$$12. \text{ ang. vel.} = 1.2\pi \text{ rad/sec}$$

$$\theta = 9.6\pi \text{ m} / 10 \text{ sec}$$

$$a) \omega = \frac{1.2\pi \text{ rad}}{\text{sec}} \times \frac{60 \text{ sec}}{1 \text{ min}}$$

$$= 72\pi \frac{\text{rad}}{\text{min}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}}$$

$$= 36 \text{ rev./min}$$

$$b) \theta = \frac{9.6\pi \text{ m}}{10 \text{ sec}} \times \frac{60 \text{ sec}}{1 \text{ min}}$$

$$= 57.6\pi \text{ m/min}$$

$$\therefore \frac{57.6\pi \text{ m}}{\text{min}} \times \frac{1 \text{ min}}{36 \text{ rev}}$$

$$= 1.6\pi \text{ m/rev.}$$

$$\rightarrow C = 2\pi r$$

$$1.6\pi \text{ m} = 2\pi r$$

$$\therefore r = \frac{1.6\pi \text{ m}}{2\pi}$$

$$= 0.8 \text{ m}$$

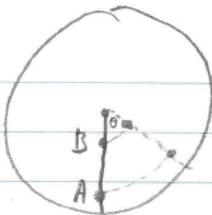
6.1 Entertainment:

pp. 320-322 #1aceg, 2aceg, 3bc, 4bc, 5, 7ab, 8ab, 9ac, 11, 12, 13.

Challenge Yourself! #10, 16* the answer for 16 should be about 86.81 radians per second.

- p.322 **13.** Two pieces of mud are stuck to the spoke of a bicycle wheel. Piece A is closer to the circumference of the tire, while piece B is closer to the centre of the wheel.
- T**
- Is the angular velocity at which piece A is travelling greater than, less than, or equal to the angular velocity at which piece B is travelling?
 - Is the velocity at which piece A is travelling greater than, less than, or equal to the velocity at which piece B is travelling?
 - If the angular velocity of the bicycle wheel increased, would the velocity at which piece A is travelling as a percent of the velocity at which piece B is travelling increase, decrease, or stay the same?

13.



$$a) \omega = \frac{\theta}{t}$$

in both cases, $\theta = 1$ radian,

\therefore the angular velocity of each piece of mud is equal.

b) $v = \frac{d}{t}$, in this case, the distance piece A travels is greater than piece B, so velocity $A >$ velocity B

$$c) v_A = \frac{2\pi r_A}{t}$$

$$v_B = \frac{2\pi r_B}{t}$$

$$\% \frac{v_A}{v_B} = \frac{2\pi r_A}{t} \div \frac{2\pi r_B}{t}$$

$$= \frac{2\pi r_A}{t} \times \frac{t}{2\pi r_B}$$

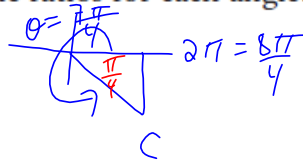
$$= \frac{r_A}{r_B} \therefore \text{constant (ie) independent of ang. velocity}$$

6.2 Entertainment: pp.330-332 #2ab, 3, 5acdf, 6cdef, 7ad, 11, 13, 15, 16, 19
 p.330

3. Determine the primary trigonometric ratios for each angle.

a) $-\frac{\pi}{2}$

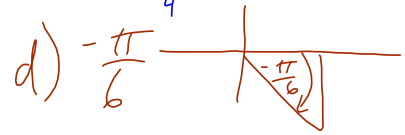
c) $\frac{7\pi}{4}$



$\beta = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

b) $-\pi$

d) $-\frac{\pi}{6}$



$\beta = \frac{\pi}{6}$

3c) $\sin \frac{7\pi}{4}$

$\cos \frac{7\pi}{4}$

$\tan \frac{7\pi}{4}$

$= -\sin(\frac{\pi}{4})$

$= +\cos(\frac{\pi}{4})$

$= -\tan(\frac{\pi}{4})$

$= -\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2}$

$= -1$

$\sin(-\frac{\pi}{6})$

$\cos(-\frac{\pi}{6})$

$\tan(-\frac{\pi}{6})$

$= -\sin(\frac{\pi}{6})$

$= +\cos(\frac{\pi}{6})$

$= -\tan(\frac{\pi}{6})$

$= -\frac{1}{2}$

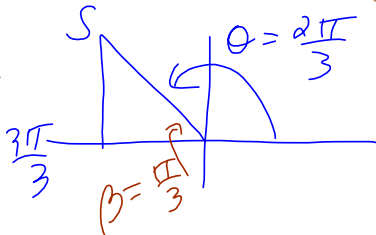
$= \frac{\sqrt{3}}{2}$

$= -\frac{\sqrt{3}}{3}$

p.330

5. Determine the exact value of each trigonometric ratio.

a) $\sin \frac{2\pi}{3}$

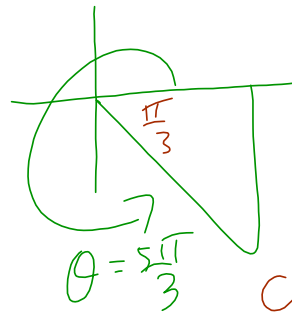


f) $\sec \frac{5\pi}{3}$

$= +\sec(\frac{\pi}{3})$

$= +\sin(\frac{\pi}{3})$

$= \frac{\sqrt{3}}{2}$



$\frac{6\pi}{3} = 2\pi$

$\beta = \frac{\pi}{3}$

$\sec(\frac{\pi}{3})$

$= \frac{1}{\cos(\frac{\pi}{3})}$

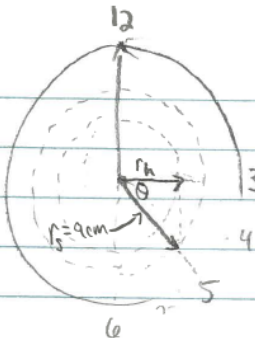
$= \frac{1}{\frac{1}{2}}$

$\therefore \sec \frac{5\pi}{3} = 2$

6.2 Entertainment: pp.330-332 #2ab, 3, 5acdf, 6cdef, 7ad, 11, 13, 15, 16, 19

- p.331 11. A clock is showing the time as exactly 3:00 p.m. and 25 s. Because a full minute has not passed since 3:00, the hour hand is pointing directly at the 3 and the minute hand is pointing directly at the 12. If the tip of the second hand is directly below the tip of the hour hand, and if the length of the second hand is 9 cm, what is the length of the hour hand?

11.

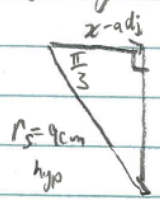


length second hand
 $= 9 \text{ cm}$
 $= r_s$

length of hour hand

blw the 3 & 5
 in 10 sec

$\therefore \theta = \frac{10}{60}$ of the central angle
 $\therefore \theta = \frac{10}{60}$ of 2π
 $= \frac{\pi}{3}$
 ≈ 1.047
 $\approx 1.05 \text{ rad}$



$\cos \frac{\pi}{3} = \frac{x}{9}$
 $x = 9 \cos \left(\frac{\pi}{3} \right)$
 $= 9 \left(\frac{1}{2} \right)$
 $= 4.5$
 (book says \approx , but really =)

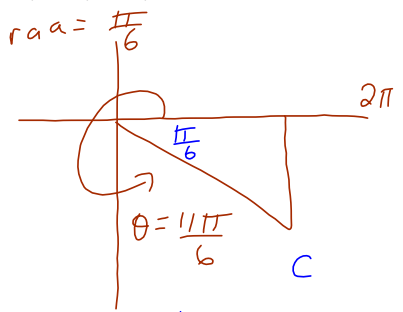
6.2 Entertainment: pp.330-332 #2ab, 3, 5acdf, 6cdef, 7ad, 11, 13, 15, 16, 19

p.332 15. Show that $2 \sin^2 \theta - 1 = \sin^2 \theta - \cos^2 \theta$ for $\frac{11\pi}{6}$. \rightarrow $\text{raa} = \frac{\pi}{6}$

$$\begin{aligned}
 LS &= 2 \sin^2 \theta - 1 & RS &= \sin^2 \theta - \cos^2 \theta \\
 &= 2 \sin^2 \left(\frac{11\pi}{6} \right) - 1 & &= \left(-\frac{1}{2} \right)^2 - \left(\frac{\sqrt{3}}{2} \right)^2 \\
 &= 2 \left(-\frac{1}{2} \right)^2 - 1 & &= \frac{1}{4} - \frac{3}{4} \\
 &= 2 \left(\frac{1}{4} \right) - 1 & &= -\frac{2}{4} \\
 &= \frac{1}{2} - 1 & &= -\frac{1}{2} \\
 &= -\frac{1}{2}
 \end{aligned}$$

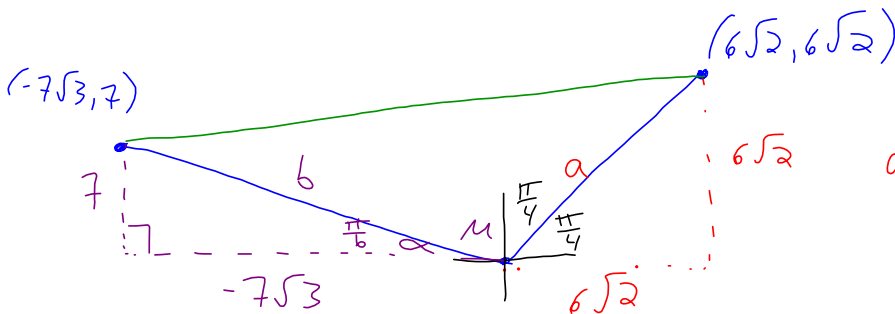
$$\therefore LS = RS$$

\therefore QED.



$$\begin{aligned}
 \sin \frac{11\pi}{6} &= -\sin \left(\frac{\pi}{6} \right) \\
 &= -\frac{1}{2}
 \end{aligned}
 \quad \left. \begin{aligned}
 \cos \frac{11\pi}{6} \\
 = + \cos \frac{\pi}{6} \\
 = \frac{\sqrt{3}}{2}
 \end{aligned} \right\}$$

p.332 19. Find the cosine of the angle formed by two rays that start at the origin of the Cartesian plane if one ray passes through the point $(6\sqrt{2}, 6\sqrt{2})$ and the other ray passes through the point $(-7\sqrt{3}, 7)$. Round your answer to the nearest hundredth, if necessary.



$$\begin{aligned}
 a^2 &= (6\sqrt{2})^2 + (6\sqrt{2})^2 \\
 &= 72 + 72 \\
 &= 144 \\
 \therefore a &= 12
 \end{aligned}$$

$$b^2 = (-7\sqrt{3})^2 + 7^2 \quad \tan \alpha = \frac{7}{-7\sqrt{3}}$$

$$= 147 + 49$$

$$= 196$$

$$\therefore b = 14$$

$$= -\frac{1}{\sqrt{3}}$$

$$\therefore \alpha = \frac{\pi}{6}$$

$$\therefore \mu = \frac{\pi}{3}$$

$$\begin{aligned}
 \text{Now } \frac{\pi}{3} + \frac{\pi}{4} \\
 &= \frac{4\pi}{12} + \frac{3\pi}{12} \\
 &= \frac{7\pi}{12}
 \end{aligned}$$

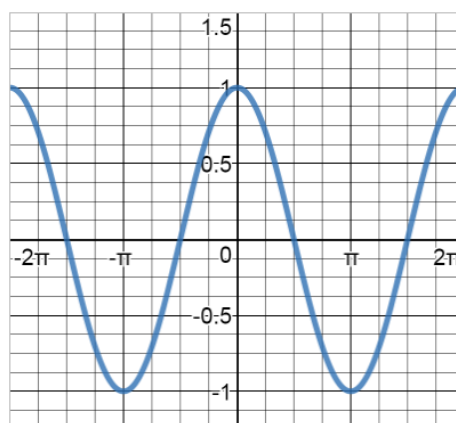
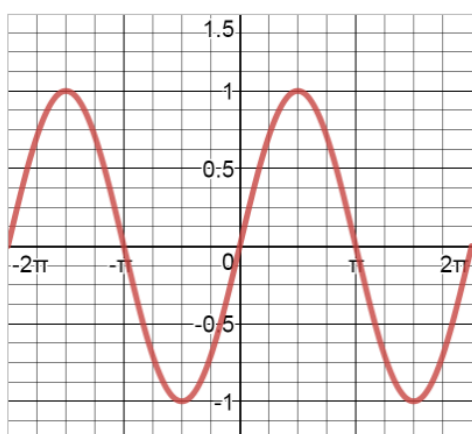
$$\therefore \cos \left(\frac{7\pi}{12} \right)$$

$$\approx -0.258$$

$$\approx -0.26$$

6.3 Entertainment: p. 336 #2c, 3, 5

- p.336 2. a) Use a graphing calculator, in radian mode, to create the graphs of the trigonometric functions $y = \sin \theta$ and $y = \cos \theta$ on the interval $-2\pi \leq \theta \leq 2\pi$. To do this, enter the functions $Y1 = \sin \theta$ and $Y2 = \cos \theta$ in the equation editor, and use the window settings shown.
- b) Determine the values of θ where the functions intersect.
- c) The equation $t_n = a + (n - 1)d$ can be used to represent the general term of any arithmetic sequence, where a is the first term and d is the common difference. Use this equation to find an expression that describes the location of each of the following values for $y = \sin \theta$, where $n \in \mathbf{I}$ and θ is in radians.
- θ -intercepts
 - maximum values
 - minimum values



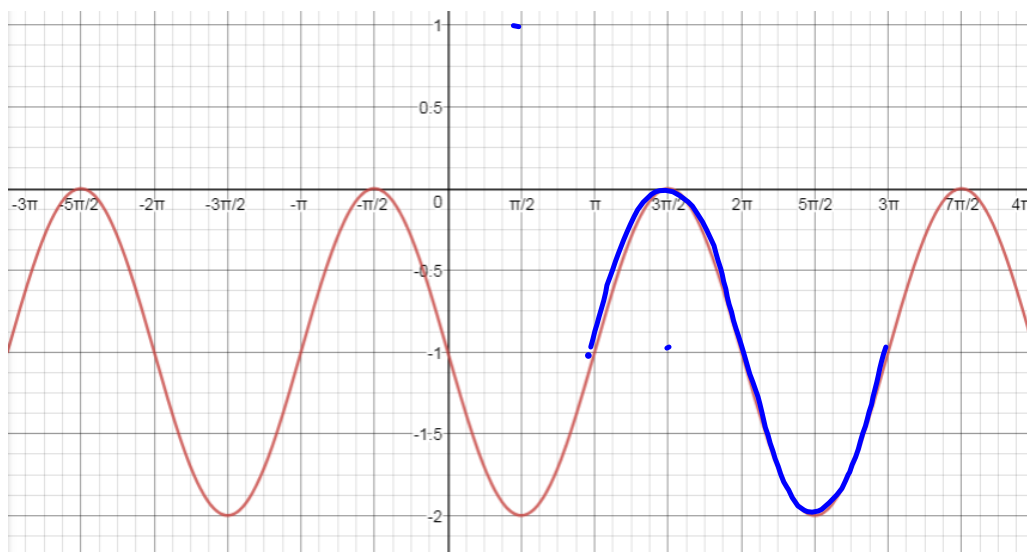
- i) $\{x \in \mathbb{R} \mid 0 + k\pi, k \in \mathbb{I}\}$
- (i) $\{x \in \mathbb{R} \mid \frac{\pi}{2} + 2k\pi, k \in \mathbb{I}\}$
- (ii) $\{x \in \mathbb{R} \mid \frac{3\pi}{2} + 2k\pi, k \in \mathbb{I}\}$

6.4 Entertainment: pp.343-346 #1ad, 4bc, 5ac, 6c, 7bc, 8c*d* graph (do not sketch),
9, 10* graph (do not sketch), 11, 12, 14a

p.345 6. State the transformations that were applied to the parent function
 $f(x) = \sin x$ to obtain each of the following transformed functions.
Then graph the transformed functions.

c) $f(x) = \sin(x - \pi) - 1$

phase shift π radians to the right
v.t. down 1 unit



6.4 Entertainment: pp.343-346 #1ad, 4bc, 5ac, 6c, 7bc, 8c*d* graph (do not sketch),
9, 10* graph (do not sketch), 11, 12, 14a

p.345 9. Each person's blood pressure is different, but there is a range of blood pressure values that is considered healthy. The function

A $P(t) = -20 \cos \frac{5\pi}{3}t + 100$ models the blood pressure, p , in millimetres of mercury, at time t , in seconds, of a person at rest.

- What is the period of the function? What does the period represent for an individual?
- How many times does this person's heart beat each minute?
- Sketch the graph of $y = P(t)$ for $0 \leq t \leq 6$.
- What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.

$$a) \text{ period} = \frac{2\pi}{\left|\frac{5\pi}{3}\right|}$$

$$= 2\pi \div \frac{5\pi}{3}$$

$$= 2\pi \times \frac{3}{5\pi}$$

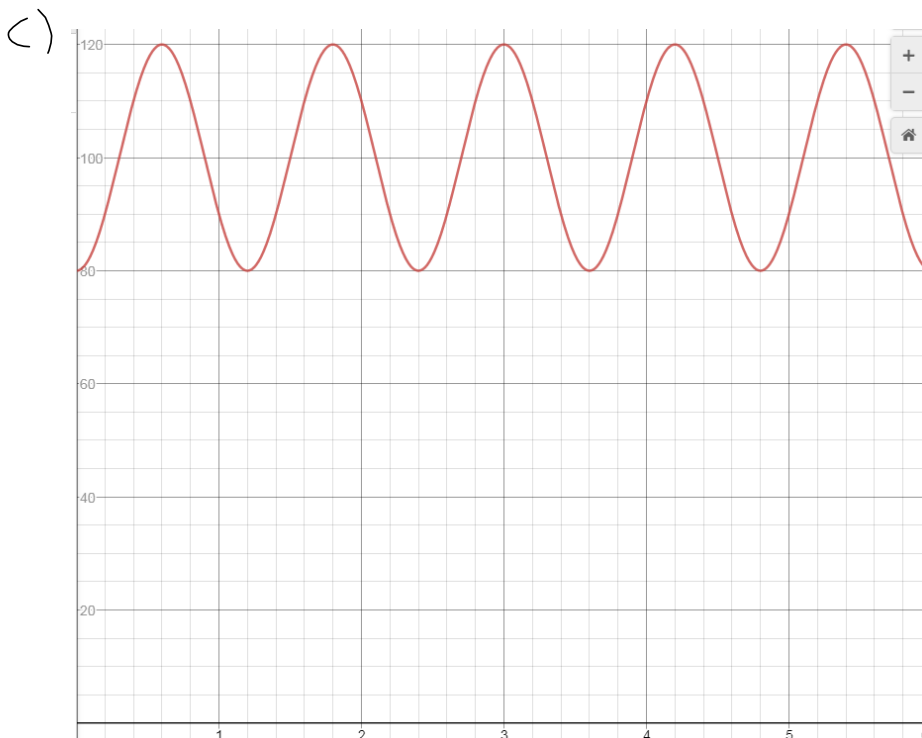
$$= \frac{6}{5}$$

b) beats per minute

$$= \frac{60}{1.2}$$

$$= 50 \text{ bpm}$$

\therefore the time between heartbeats is $1\frac{1}{5}$ seconds (or 1.2 sec).



d) Range:

$$[80, 120]$$

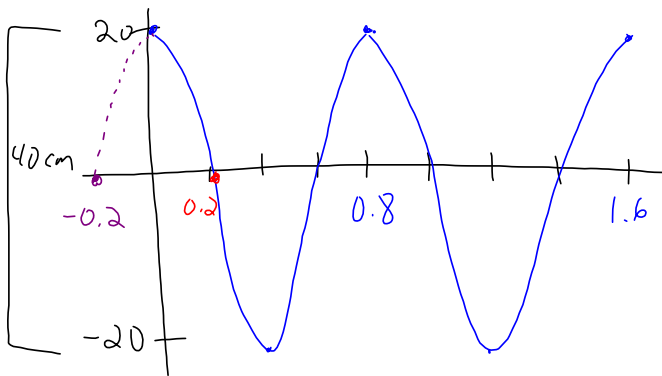
\therefore blood pressure ranges from

80 mm Hg to 120 mm of Hg

6.4 Entertainment: pp.343-346 #1ad, 4bc, 5ac, 6c, 7bc, 8c*d* graph (do not sketch),
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p.345 10. A pendulum swings back and forth 10 times in 8 s. It swings through a total horizontal distance of 40 cm.

- Sketch a graph of this motion for two cycles, beginning with the pendulum at the end of its swing.
- Describe the transformations necessary to transform $y = \sin x$ into the function you graphed in part a).
- Write the equation that models this situation.



$$\text{period} = \frac{8 \text{ sec}}{10 \text{ cycles}} \\ = 0.8 \text{ sec/cycle}$$

$$\therefore y = 20 \cos\left(\frac{5\pi}{2}(x-0)\right) + 0$$

but question asks for sine!

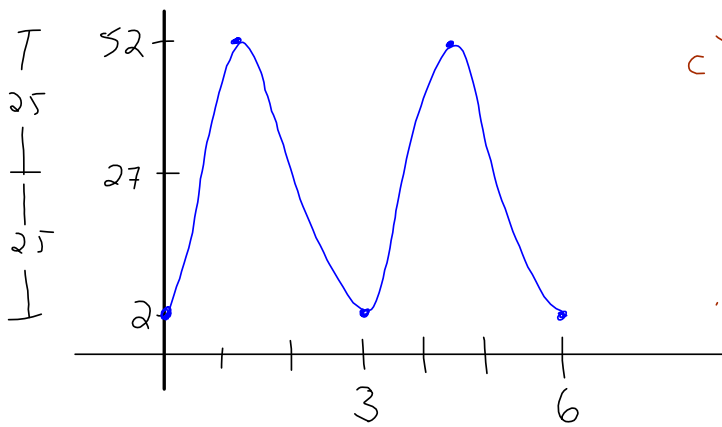
$$\therefore y = -20 \sin\left(\frac{5\pi}{2}(x-0.2)\right)$$

$$\text{or } y = 20 \sin\left(\frac{5\pi}{2}(x+0.2)\right)$$

$$\begin{aligned} \text{c) } a &= 20 \\ k &= \frac{\text{original period}}{\text{period}} \\ &= \frac{2\pi}{0.8} \\ &= 2\pi \div \frac{8}{10} \\ &= 2\pi \times \frac{10}{8} \\ &= \frac{5\pi}{2} \end{aligned}$$

6.4 Entertainment: pp.343-346 #1ad, 4bc, 5ac, 6c, 7bc, 8c*d* graph (do not sketch),
9, 10* graph (do not sketch), 11, 12, 14a

- p.346 11. A rung on a hamster wheel, with a radius of 25 cm, is travelling at a constant speed. It makes one complete revolution in 3 s. The axle of the hamster wheel is 27 cm above the ground.
- T**
- Sketch a graph of the height of the rung above the ground during two complete revolutions, beginning when the rung is closest to the ground.
 - Describe the transformations necessary to transform $y = \cos x$ into the function you graphed in part a).
 - Write the equation that models this situation.



$$c) a = 25 \quad k = \frac{2\pi}{3}$$

$$d = 0, \quad c = 27$$

$$\begin{aligned} \therefore y &= -25 \cos\left(\frac{2\pi}{3}(x-0)\right) + 27 \\ &= -25 \cos\left(\frac{2\pi}{3}x\right) + 27 \end{aligned}$$