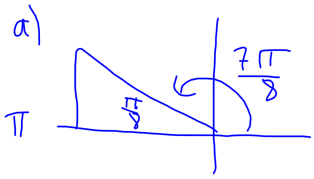
Last Day's Entertainment: pp.392-393 #1, 3abc, 5abc, 7

5. Write an expression that is equivalent to each of the following expressions, using the related acute angle.

a) $\sin \frac{7\pi}{8}$ **c)** $\tan \frac{5\pi}{4}$ **e)** $\sin \frac{13\pi}{8}$

b) $\cos \frac{13\pi}{12}$ **d)** $\cos \frac{11\pi}{6}$ **f)** $\tan \frac{5\pi}{3}$

a)

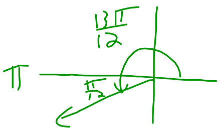


$\sin \frac{7\pi}{8}$

$= + \sin \frac{\pi}{8}$

$= \sin \frac{\pi}{8}$

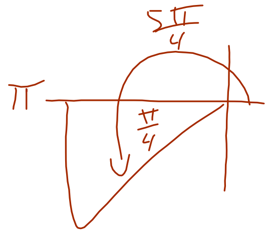
b) $\cos \frac{13\pi}{12}$



$\cos \frac{13\pi}{12}$

$= -\cos\left(\frac{\pi}{12}\right)$

c) $\tan \frac{5\pi}{4}$



$= + \tan\left(\frac{\pi}{4}\right)$

$= \tan \frac{\pi}{4}$

or ?

a) $\sin \frac{7\pi}{8}$

$= \cos\left(\frac{\pi}{2} - \frac{7\pi}{8}\right)$

$= \cos\left(\frac{4\pi}{8} - \frac{7\pi}{8}\right)$

$= \cos\left(-\frac{3\pi}{8}\right)$

$= \cos\left(\frac{3\pi}{8}\right)$

7.2 Compound Angle Formulas (Day 1)

"By the end of next class' lesson:
I can derive and apply all compound angle formulas.
I can apply what I have learned in unfamiliar settings."

Trig Identities from last class...

Odd and Even

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(*see summary on p. 391*)

Cofunction

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$

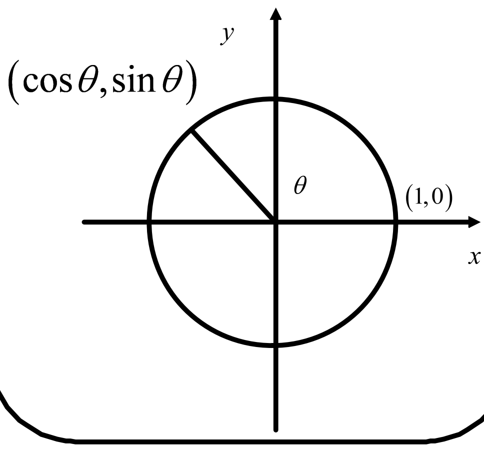
(*see summary on p. 392*)

On a circle with radius *r*...

$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$

"SYR CXR TYX"

This means if $r = 1$ then
$\sin\theta = y, \quad \cos\theta = x$



Recall:

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ → $\sin^2\theta = 1 - \cos^2\theta$, $\cos^2\theta = 1 - \sin^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

A sneak-preview of the
Compound Angle Identities:

Addition Formulas

$\sin(a+b) = \sin a\cos b + \cos a\sin b$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a\tan b}$

Subtraction Formulas

$\sin(a-b) = \sin a\cos b - \cos a\sin b$

$\cos(a-b) = \cos a\cos b + \sin a\sin b$

$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a\tan b}$

A **compound angle** is an angle that is created by adding or subtracting two or more angles.

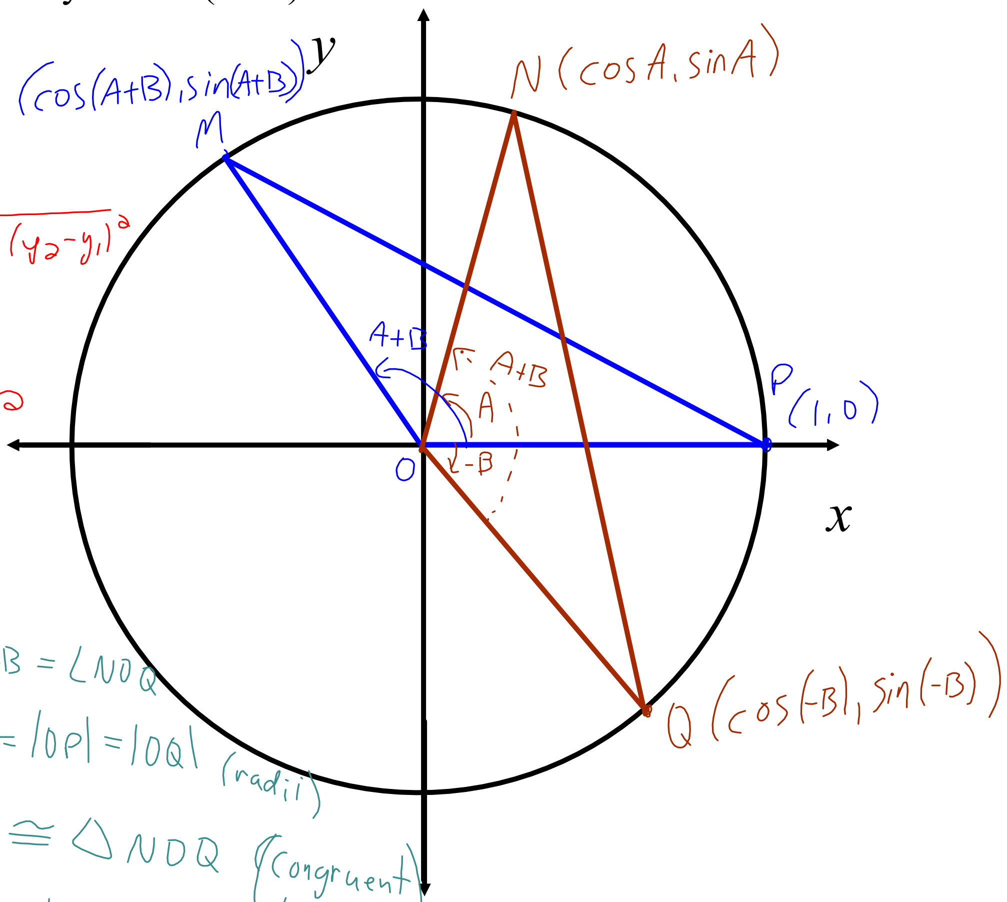
Create an identity for $\cos(a + b)$

Recall:

$l=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$(x+y)^2$

$=x^2+2xy+y^2$



$\angle MOP = A+B = \angle NOQ$

$|OM| = |ON| = |OP| = |OQ|$ (radii)

$\therefore \triangle MOP \cong \triangle NOQ$ (congruent triangles)

$\therefore |MP| = |NQ|$

$|MP| = |NQ|$ * square both sides

$$\sqrt{(\cos(A+B)-1)^2+(\sin(A+B)-0)^2}=\sqrt{(\cos A-\cos(-B))^2+(\sin A-\sin(-B))^2}$$

$$\cos^2(A+B)-2\cos(A+B)+1+\sin^2(A+B)=\cos^2A-2\cos A\cos(-B)+\cos^2(-B)+\sin^2A-2\sin A\sin(-B)+\sin^2(-B)$$

$$\underbrace{\cos^2(A+B)+\sin^2(A+B)}+1-2\cos(A+B)=\underbrace{\cos^2A+\sin^2A}+\underbrace{\sin^2(-B)+\cos^2(-B)}-2\cos A\cos(-B)-2\sin A\sin(-B)$$

$$1+1-2\cos(A+B) = 1 + 1 - 2\cos A\cos(-B)-2\sin A\sin(-B)$$

$$\therefore -2\cos(A+B) = -2\cos A\cos(-B)-2\sin A\sin(-B)$$

* cos is even
* sin is odd

$$= -2\cos A\cos B + 2\sin A\sin B \qquad \div(-2)$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

Also $\cos(A-B) = \cos A\cos B + \sin A\sin B$

Now let's create an identity for $\sin(a + b)$

Recall: $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$

$$\sin(A+B)$$
$$= \cos\left(\frac{\pi}{2} - (A+B)\right)$$
$$= \cos\left(\frac{\pi}{2} - A - B\right)$$
$$= \cos\left(\left(\frac{\pi}{2} - A\right) - B\right)$$

$* \cos(A-B) = \cos A \cos B + \sin A \sin B$

$$= \cos\left(\frac{\pi}{2} - A\right)\cos B + \sin\left(\frac{\pi}{2} - A\right)\sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

Also $\sin(A-B) = \sin A \cos B - \cos A \sin B$

Ex. 1: Simplify to an exact value:

$$\cos\frac{5\pi}{12}\cos\frac{7\pi}{12} + \sin\frac{5\pi}{12}\sin\frac{7\pi}{12}$$

$$= \cos\left(\frac{5\pi}{12} - \frac{7\pi}{12}\right)$$
$$= \cos\left(\frac{-2\pi}{12}\right)$$
$$= \cos\left(\frac{-\pi}{6}\right)$$
$$= \cos\left(\frac{\pi}{6}\right)$$
$$= \frac{\sqrt{3}}{2}$$

Ex. 2: Find the exact value of $\sin\frac{\pi}{12}$

$$\frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12}$$
$$= \frac{\pi}{4} - \frac{\pi}{6}$$
$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$
$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$\left(\frac{\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$$

Entertainment...
pp.400-401 #1, 2b, 3, 4ade, 5abd, 6ace, 7ace, 8cd, 11, 16