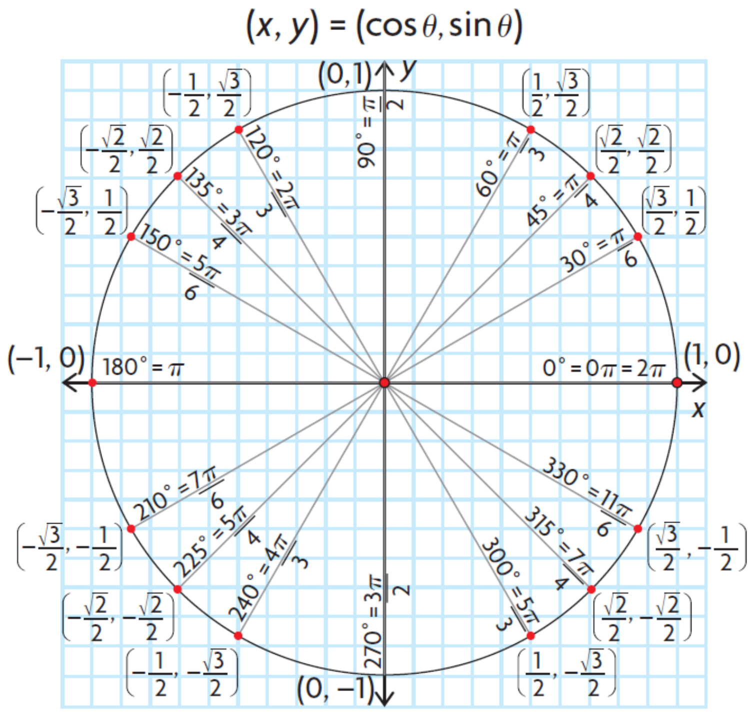

(x, y) = (cos θ, sin θ)
(0,1)
y
x
(1, 0)
(−1, 0)
(0, −1)
0° = 0π = 2π
30° = π/6
45° = π/4
60° = π/3
90° = π/2
120° = 2π/3
135° = 3π/4
150° = 5π/6
180° = π
210° = 7π/6
225° = 5π/4
240° = 4π/3
270° = 3π/2
300° = 5π/3
315° = 7π/4
330° = 11π/6
(√3/2, 1/2)
(√2/2, √2/2)
(1/2, √3/2)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(√2/2, −√2/2)
(√3/2, −1/2)

Entertainment...

pp.400-401 #1, 2b, 3, 4ade, 5abd, 6ace, 7ace, 8cd, 11, 16

a d

p.400

1. Rewrite each expression as a single trigonometric ratio.
   a) $\sin a \cos 2a + \cos a \sin 2a$

$\sin(a + 2a)$

$= \sin(3a)$

3. Express each angle as a compound angle, using a pair of angles from the special triangles.

a) $75°$ c) $-\frac{\pi}{6}$ e) $105°$

b) $-15°$ d) $\frac{\pi}{12}$ f) $\frac{5\pi}{6}$

d) $\frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12}$

$= \frac{\pi}{4} - \frac{\pi}{6}$

Entertainment...
pp.400-401 #1, 2b, 3, 4ade, 5abd, 6ace, 7ace, 8cd, 11, 16
a d

p.400

4. Determine the exact value of each trigonometric ratio.

a) $\sin 75°$ c) $\tan \frac{5\pi}{12}$ e) $\cos 105°$

b) $\cos 15°$ d) $\sin\left(-\frac{\pi}{12}\right)$ f) $\tan \frac{23\pi}{12}$

$30° \; \frac{\pi}{6}$
$45 \; \frac{\pi}{4} \; \frac{\pi}{12}$
$60° \; \frac{\pi}{3} \; \frac{4\pi}{12}$
* $90° \; \frac{\pi}{2} \; \frac{6\pi}{12}$
$120° \; \frac{2\pi}{3} \; \frac{8\pi}{12}$
$135° \; \frac{3\pi}{4} \; \frac{9\pi}{12}$
$150° \; \frac{5\pi}{6} \; \frac{10\pi}{12}$
$180° \; \pi \; \frac{12\pi}{12}$

a) $\sin 75°$
$= \sin(30°+45°)$
$= \sin 30°\cos 45° + \cos 30°\sin 45°$
$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

d) $\sin\left(-\frac{\pi}{12}\right)$
$= \sin\left(\frac{2\pi}{12} - \frac{3\pi}{12}\right)$
$= \sin\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$
$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} - \cos\frac{\pi}{6}\sin\frac{\pi}{4}$
$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \sin\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)$
$= \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$
$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3}$
$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$
$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

e) $\cos 105°$
$= \cos(60°+45°)$
$= \cos 60°\cos 45° - \sin 60°\sin 45°$
$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

5. Use the appropriate compound angle formula to determine the exact value of each expression.

a) $\sin\left(\pi + \frac{\pi}{6}\right)$ c) $\tan\left(\frac{\pi}{4} + \pi\right)$ e) $\tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$

b) $\cos\left(\pi - \frac{\pi}{4}\right)$ d) $\sin\left(-\frac{\pi}{2} + \frac{\pi}{3}\right)$ f) $\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

d) $\sin\left(-\frac{\pi}{2} + \frac{\pi}{3}\right)$
$= \sin\left(-\frac{\pi}{2}\right)\cos\frac{\pi}{3} + \cos\left(-\frac{\pi}{2}\right)\sin\frac{\pi}{3}$
$= (-1)\left(\frac{1}{2}\right) + (0)\left(\frac{\sqrt{3}}{2}\right)$
$= -\frac{1}{2}$

a) $\sin\left(\pi + \frac{\pi}{6}\right)$
$= \sin\pi\cos\frac{\pi}{6} + \cos\pi\sin\frac{\pi}{6}$
$= (0)\left(\frac{\sqrt{3}}{2}\right) + (-1)\left(\frac{1}{2}\right)$
$= -\frac{1}{2}$

b) $\cos\left(\pi - \frac{\pi}{4}\right)$
$= \cos\pi\cos\frac{\pi}{4} + \sin\pi\sin\frac{\pi}{4}$
$= (-1)\left(\frac{\sqrt{2}}{2}\right) + (0)\left(\frac{\sqrt{2}}{2}\right)$
$= -\frac{\sqrt{2}}{2}$

p.401 **8.** Determine the exact value of each trigonometric ratio.

a) $\cos 75°$ c) $\cos \frac{11\pi}{12}$ e) $\tan \frac{7\pi}{12}$

b) $\tan(-15°)$ d) $\sin \frac{13\pi}{12}$ f) $\tan \frac{-5\pi}{12}$

c) $\cos \frac{11\pi}{12}$

$= \cos\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right)$

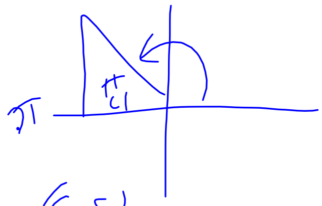
$= \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{-\sqrt{2}(\sqrt{3}+1)}{4}$



d) $\sin\frac{13\pi}{12}$

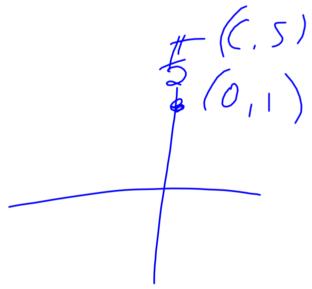
$= \sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$= \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$



11. Use compound angle formulas to verify each of the following cofunction identities.

a) $\sin x = \cos\left(\frac{\pi}{2} - x\right)$ b) $\cos x = \sin\left(\frac{\pi}{2} - x\right)$

$RS = \cos\left(\frac{\pi}{2} - x\right)$

$= \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= (0)\cos x + (1)\sin x$

$= \sin x$

$= LS$

$\therefore LS = RS$

$\therefore$ QED!

b) $RS = \sin\left(\frac{\pi}{2} - x\right)$

$= \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$

$= (1)\cos x - (0)(\sin x)$

$= \cos x$

$= LS$

$\therefore LS = RS$

$\therefore$ QED!

p.401 **16.** Prove $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$.

$RS = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$= 2\left[\sin\left(\frac{C}{2}+\frac{D}{2}\right)\cos\left(\frac{C}{2}-\frac{D}{2}\right)\right]$

$= 2\left[\sin\left(\frac{C}{2}\right)\cos\left(\frac{D}{2}\right) + \cos\left(\frac{C}{2}\right)\sin\left(\frac{D}{2}\right)\right]\left[\cos\left(\frac{C}{2}\right)\cos\left(\frac{D}{2}\right) + \sin\left(\frac{C}{2}\right)\sin\left(\frac{D}{2}\right)\right]$

$= 2\left(\cos^2\left(\frac{D}{2}\right)\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right) + \sin^2\left(\frac{C}{2}\right)\sin\left(\frac{D}{2}\right)\cos\left(\frac{D}{2}\right) + \cos^2\left(\frac{C}{2}\right)\sin\left(\frac{D}{2}\right)\cos\left(\frac{D}{2}\right) + \sin^2\left(\frac{D}{2}\right)\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)\right)$

$= 2\left(\cos^2\frac{D}{2}\sin\frac{C}{2}\cos\frac{C}{2} + \sin^2\frac{D}{2}\sin\frac{C}{2}\cos\frac{C}{2} + \sin^2\frac{C}{2}\sin\frac{D}{2}\cos\frac{D}{2} + \cos^2\frac{C}{2}\sin\frac{D}{2}\cos\frac{D}{2}\right)$

$= 2\left(\sin\frac{C}{2}\cos\frac{C}{2}\underbrace{\left(\cos^2\frac{D}{2}+\sin^2\frac{D}{2}\right)}_{1} + \sin\frac{D}{2}\cos\frac{D}{2}\underbrace{\left(\sin^2\frac{C}{2}+\cos^2\frac{C}{2}\right)}_{1}\right)$

$= 2\left(\sin\frac{C}{2}\cos\frac{C}{2} + \sin\frac{D}{2}\cos\frac{D}{2}\right)$

* Check LS when can't see next step

$LS = \sin C + \sin D$

$= \sin\left(\frac{C}{2}+\frac{C}{2}\right) + \sin\left(\frac{D}{2}+\frac{D}{2}\right)$

$= \sin\frac{C}{2}\cos\frac{C}{2} + \cos\frac{C}{2}\sin\frac{C}{2} + \sin\frac{D}{2}\cos\frac{D}{2} + \cos\frac{D}{2}\sin\frac{D}{2}$

$= 2\sin\frac{C}{2}\cos\frac{C}{2} + 2\sin\frac{D}{2}\cos\frac{D}{2}$

$= 2\left(\sin\frac{C}{2}\cos\frac{C}{2} + \sin\frac{D}{2}\cos\frac{D}{2}\right)$

$\therefore LS = RS$ (See RS above)

$\therefore$ QED !!!

7.2 Compound Angle Formulas (Day 2)

"By the end of today's lesson:
I can derive and apply all compound angle formulas.
I can apply what I have learned in unfamiliar settings."

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

Note: x + y are angles, NOT coordinates

Ex. 1: If $\sin x = \frac{2}{3}$ and $\cos y = -\frac{4}{5}$, where $\frac{\pi}{2} < x < \pi$, $\frac{\pi}{2} < y < \pi$,

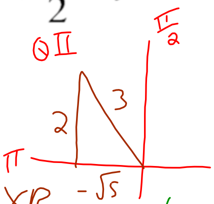
evaluate $\sin(x - y)$

$= \sin x \cos y - \cos x \sin y$

$= \left(\frac{2}{3}\right)\left(\frac{-4}{5}\right) - \left(\frac{-\sqrt{5}}{3}\right)\left(\frac{3}{5}\right)$

$= \frac{-8}{15} + \frac{3\sqrt{5}}{15}$

$= \frac{-8+3\sqrt{5}}{15}$



SYR | CXR

$\sin = \frac{y}{r}$ | $\cos = \frac{x}{r}$

$= \frac{2}{3}$

$x^2 + y^2 = r^2$

$x^2 + 2^2 = 3^2$

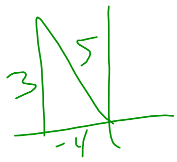
$x^2 = 9 - 4$

$x = \pm\sqrt{5}$

but in QII

$x = -\sqrt{5}$

$\therefore \cos x = \frac{-\sqrt{5}}{3}$



$\cos = \frac{-4}{5}$

$\therefore x = -4, r = 5$

$(-4)^2 + y^2 = 5^2$

$y^2 = 25 - 16$

$= 9$

$\therefore y = \pm 3$

but in QII

$y = 3$

$\sin y = \frac{3}{5}$

Ex. 2: Prove the identity $\tan(a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$LS = \tan(A+B)$

$$= \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \div \frac{\cos A}{\cos A}$$

$$= \frac{\frac{\sin A \cos B}{\cos A} + \frac{\cancel{\cos A} \sin B}{\cancel{\cos A}}}{\frac{\cancel{\cos A} \cos B}{\cancel{\cos A}} - \frac{\sin A \sin B}{\cos A}}$$

$$= \frac{\tan A \cos B + \sin B}{\cos B - \tan A \sin B} \div \frac{\cos B}{\cos B}$$

$$= \frac{\frac{\tan A \cancel{\cos B}}{\cancel{\cos B}} + \frac{\sin B}{\cos B}}{\frac{\cancel{\cos B}}{\cancel{\cos B}} - \frac{\tan A \sin B}{\cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$= RS$

$\therefore LS = RS$

$\therefore$ QED!

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$RS = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}}{1 + \frac{\sin A}{\cos A} \times \frac{\sin B}{\cos B}}$$

$$= \frac{\frac{\sin A \cos B - \sin B \cos A}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} + \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\sin A \cos B - \sin B \cos A}{\cos A \cos B} \div \frac{\cos A \cos B + \sin A \sin B}{\cos A \cos B}$$

$$= \frac{\sin A \cos B - \sin B \cos A}{\cos A \cos B} \times \frac{\cos A \cos B}{\cos A \cos B + \sin A \sin B}$$

$$= \frac{\sin A \cos B - \sin B \cos A}{\cos A \cos B + \sin A \sin B}$$

$$= \frac{\sin(A-B)}{\cos(A-B)}$$

$= \tan(A-B)$

$= LS$

$\therefore LS = RS$

$\therefore$ QED!

pp. 400-401 #2a, 4cf, 5ce, 6df, 7df, 8ef, 9, 10, **12**, 17