

Last day's work:

pp. 400-401 #2a, 4cf, 5ce, 6df, 7df, 8ef, 9, 10, 12, 17

p.400 4. Determine the exact value of each trigonometric ratio.

- a) $\sin 75^\circ$ c) $\tan \frac{5\pi}{12}$ e) $\cos 105^\circ$
 b) $\cos 15^\circ$ d) $\sin\left(-\frac{\pi}{12}\right)$ f) $\tan \frac{23\pi}{12}$

$$\begin{aligned} c) \tan\left(\frac{5\pi}{12}\right) &= \tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) \\ &= \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right) \\ &= \frac{\tan \frac{\pi}{6} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{6} \tan \frac{\pi}{4}} \\ &= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \left(\frac{\sqrt{3}}{3}\right)(1)} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{\sqrt{3}}{3} + \frac{3}{3}}{\frac{3 - \sqrt{3}}{3}} \\ &= \frac{\sqrt{3} + 3}{3 - \sqrt{3}} \end{aligned}$$

$$= \frac{\sqrt{3} + 3}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{\sqrt{3} + 3}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{\sqrt{3} + 3}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{3\sqrt{3} + 3 + 9 + 3\sqrt{3}}{9 + 3\sqrt{3} - 3\sqrt{3} - 3}$$

$$= \frac{12 + 6\sqrt{3}}{6}$$

$$= \frac{6(2 + \sqrt{3})}{6}$$

$$= 2 + \sqrt{3}$$

$$\begin{aligned} f) \tan \frac{23\pi}{12} &= \tan\left(\frac{14\pi}{12} + \frac{9\pi}{12}\right) \\ &= \tan\left(\frac{7\pi}{6} + \frac{3\pi}{4}\right) \\ &= \frac{\tan \frac{7\pi}{6} + \tan \frac{3\pi}{4}}{1 - \tan \frac{7\pi}{6} \tan \frac{3\pi}{4}} \\ &= \frac{\left(\frac{\sqrt{3}}{3}\right) + (-1)}{1 - \left(\frac{\sqrt{3}}{3}\right)(-1)} \end{aligned}$$

$$= \frac{\sqrt{3} - 3}{3 + \sqrt{3}}$$

$$= \frac{\sqrt{3} - 3}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{\sqrt{3} - 3}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{3\sqrt{3} - 3 - 9 + 3\sqrt{3}}{9 - 3}$$

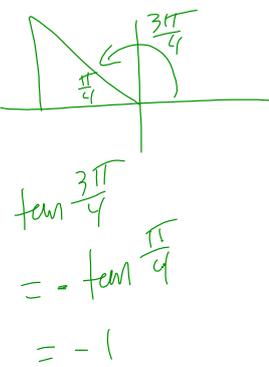
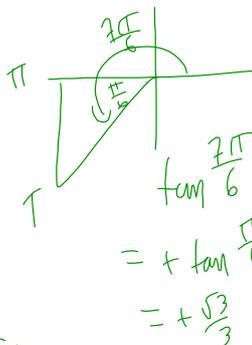
$$= \frac{-12 + 6\sqrt{3}}{6}$$

$$= \frac{6(-2 + \sqrt{3})}{6}$$

$$= -2 + \sqrt{3}$$

*Note: $\frac{\sqrt{3} + 3}{3 - \sqrt{3}}$ is an acceptable final answer.

| | |
|------------------|--------------------|
| $\frac{\pi}{6}$ | $\frac{2\pi}{12}$ |
| $\frac{\pi}{4}$ | $\frac{3\pi}{12}$ |
| $\frac{\pi}{3}$ | $\frac{4\pi}{12}$ |
| $\frac{\pi}{2}$ | $\frac{6\pi}{12}$ |
| $\frac{2\pi}{3}$ | $\frac{8\pi}{12}$ |
| $\frac{3\pi}{4}$ | $\frac{9\pi}{12}$ |
| $\frac{5\pi}{6}$ | $\frac{10\pi}{12}$ |
| $\frac{7\pi}{6}$ | $\frac{14\pi}{12}$ |



*Note: $\frac{\sqrt{3} - 3}{3 + \sqrt{3}}$ is an acceptable final answer.

p.401 8. Determine the exact value of each trigonometric ratio.

a) $\cos 75^\circ$

c) $\cos \frac{11\pi}{12}$

e) $\tan \frac{7\pi}{12}$

b) $\tan (-15^\circ)$

d) $\sin \frac{13\pi}{12}$

f) $\tan \frac{-5\pi}{12}$

8f) $\tan \left(\frac{-5\pi}{12} \right)$

$$= \tan \left(\frac{3\pi}{12} - \frac{8\pi}{12} \right)$$

$$= \tan \left(\frac{\pi}{4} - \frac{2\pi}{3} \right)$$

$$= \frac{\tan \frac{\pi}{4} - \tan \frac{2\pi}{3}}{1 + \tan \frac{\pi}{4} \tan \frac{2\pi}{3}}$$

$$= \frac{1 - (-\sqrt{3})}{1 + 1(-\sqrt{3})}$$

$$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

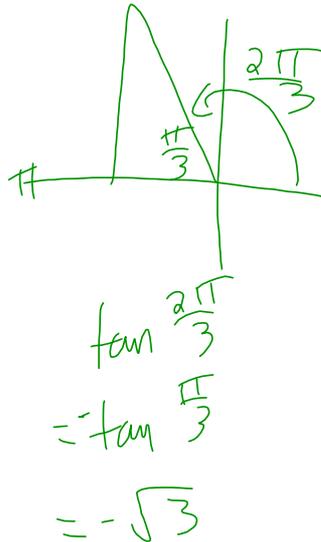
$$= \frac{1 + 2\sqrt{3} + 3}{1 - 3}$$

$$= \frac{4 + 2\sqrt{3}}{-2}$$

$$= \frac{2(2 + \sqrt{3})}{-2(-1)}$$

$$= \frac{2 + \sqrt{3}}{-1}$$

$$= -2 - \sqrt{3}$$



*Note: $\frac{1 + \sqrt{3}}{1 - \sqrt{3}}$ is an acceptable final answer.

7.3 Double Angle Formulas



"By the end of today's lesson:

I can derive and apply all double angle identities.

I can apply what I have learned in unfamiliar settings."

Recall: Compound Angle Identities

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Use the compound angle for sine to develop the double angle identity for

$$\sin 2A$$

$$= \sin(A+A)$$

$$= \sin A \cos A + \cos A \sin A$$

$$= \sin A \cos A + \sin A \cos A$$

$$= 2 \sin A \cos A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\cos^2 A = 1 - \sin^2 A$$

Use the compound angle for cosine to develop the double angle identity for

$$\cos 2A$$

$$= \cos(A+A)$$

$$= \cos A \cos A - \sin A \sin A$$

$$= \cos^2 A - \sin^2 A$$

$$= \cos^2 A - (1 - \cos^2 A)$$

$$= \cos^2 A - 1 + \cos^2 A$$

$$= 2 \cos^2 A - 1$$

$$= (1 - \sin^2 A) - \sin^2 A$$

$$= 1 - 2 \sin^2 A$$

Use the compound angle for tangent to develop the double angle identity for

$$\tan 2A$$

$$= \tan(A+A)$$

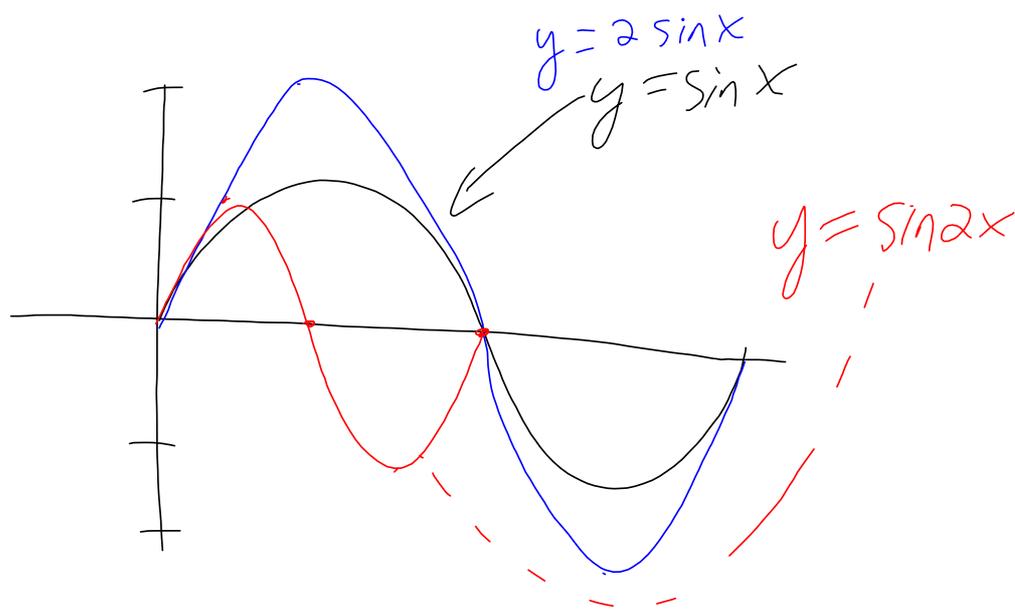
$$= \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$= \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$



Ex. 1: If $\sin \theta = -\frac{2}{5}$ for $0 \leq \theta \leq 2\pi$ then find $\cos 2\theta$ and $\sin 2\theta$.

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$$y = -2, r = 5$$

$$x^2 + (-2)^2 = 5^2$$

$$x^2 = 25 - 4$$

$$= 21$$

$$x = \pm \sqrt{21}$$

in QIII

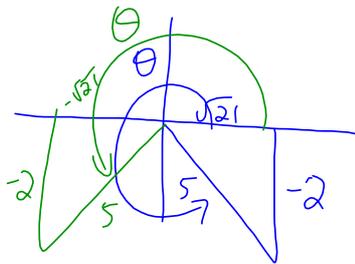
$$x = -\sqrt{21}$$

$$\cos \theta = \frac{-\sqrt{21}}{5}$$

in QIV

$$x = \sqrt{21}$$

$$\cos \theta = \frac{\sqrt{21}}{5}$$



$$\cos 2\theta$$

$$= \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{-\sqrt{21}}{5}\right)^2 - \left(-\frac{2}{5}\right)^2$$

$$= \frac{21}{25} - \frac{4}{25}$$

$$= \frac{17}{25}$$

$$\sin 2\theta$$

$$= 2 \sin \theta \cos \theta$$

in QIII

$$= 2 \left(-\frac{2}{5}\right) \left(-\frac{\sqrt{21}}{5}\right)$$

$$= \frac{4\sqrt{21}}{25}$$

in QIV

$$= 2 \left(\frac{-2}{5}\right) \left(\frac{\sqrt{21}}{5}\right)$$

$$= \frac{-4\sqrt{21}}{25}$$

pp. 407-408 #1 to 5, 9, 12, 16, 17

*the text has an incorrect answer for #3b: the right bracket should be before -1