

Last day's work: *2f, 3b, 4*

pp. 407-408 #1 to 5, 9, 12, 16, 17

*the text has an incorrect answer for #3b: the right bracket should be before -1

- p.407 2. Express each of the following as a single trigonometric ratio and then evaluate.

a) $2 \sin 45^\circ \cos 45^\circ$

d) $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$

f) $2 \tan(60^\circ) \cos^2 60^\circ$

$$= 2 \frac{\sin 60^\circ}{\cos 60^\circ} \cdot (\cos^2 60^\circ)$$

$$= 2 \sin 60^\circ \cos 60^\circ$$

$$= \sin 120^\circ$$

$$= \sin 120^\circ$$

$$= \frac{\sqrt{3}}{2}$$

b) $\cos^2 30^\circ - \sin^2 30^\circ$

e) $1 - 2 \sin^2 \frac{3\pi}{8}$

c) $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$

f) $2 \tan 60^\circ \cos^2 60^\circ$

- p.407 3. Use a double angle formula to rewrite each trigonometric ratio.

a) $\sin 4\theta$

d) $\cos 6\theta$

b) $\cos 3x$

e) $\sin x$

**book wrong*

b) $\cos 3x$

$$= \cos^2 \frac{3}{2}x - \sin^2 \frac{3}{2}x$$

$$= 2 \cos^2 \frac{3}{2}x - 1$$

$$\rightarrow \cos^2 \frac{3}{2}x - (1 - \cos^2 \frac{3}{2}x)$$

$$= \underline{\cos^2 \frac{3}{2}x} - 1 + \underline{\cos^2 \frac{3}{2}x}$$

4. Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given

K $\cos \theta = \frac{3}{5}$ and $0 \leq \theta \leq \frac{\pi}{2}$.

4. $\cos \theta = \frac{3}{5}$, in QI
 $\therefore \sin \theta = \frac{4}{5}$, $\tan \theta = \frac{4}{3}$

$\sin 2\theta = 2 \sin \theta \cos \theta$
 $= 2 \left(\frac{4}{5}\right) \left(\frac{3}{5}\right)$
 $= \frac{24}{25}$

$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
 $= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$
 $= \frac{9-16}{25}$
 $= -\frac{7}{25}$

$$\begin{aligned} \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= 2 \left(\frac{4}{3}\right) \\ &\quad \overline{1 - \left(\frac{4}{3}\right)^2} \\ &= \frac{8}{3} \div \left(1 - \frac{16}{9}\right) \\ &= \frac{8}{3} \div \left(-\frac{7}{9}\right) \\ &= \frac{8}{3} \times -\frac{9}{7} \\ &= -\frac{24}{7} \end{aligned}$$

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5. Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given

$$\tan \theta = -\frac{7}{24} \text{ and } \frac{\pi}{2} \leq \theta \leq \pi.$$

$$5. \tan \theta = -\frac{7}{24} \text{ in QII}$$

$$\therefore \sin \theta = \frac{7}{25}, \cos \theta = -\frac{24}{25}$$

$$\sin 2\theta = 2 \left(\frac{7}{25} \right) \left(-\frac{24}{25} \right)$$

$$= -\frac{336}{625}$$

$$\begin{aligned}\cos 2\theta &= 1 - 2 \sin^2 \theta \\ &= 1 - 2 \left(\frac{7}{25} \right)^2 \\ &= \frac{625}{625} - \frac{98}{625} \\ &= \frac{625 - 98}{625} \\ &= \frac{527}{625}\end{aligned}$$

$$\begin{aligned}\tan 2\theta &= \frac{2 \left(-\frac{7}{24} \right)}{1 - \left(-\frac{7}{24} \right)^2} \\ &= -\frac{7}{12} \div \left(\frac{576}{576} - \frac{49}{576} \right) \\ &= -\frac{7}{12} \times \frac{576}{527} \\ &= -\frac{336}{527}\end{aligned}$$

- p.407 9. Jim needs to find the sine of $\frac{\pi}{8}$. If he knows that $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$, how can he use this fact to find the sine of $\frac{\pi}{8}$? What is his answer?

$$9. \sin \frac{\pi}{8}; \text{ if } \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\because \cos 2x = 1 - 2\sin^2 x$$

$$\therefore \cos 2\left(\frac{\pi}{8}\right) = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\cos \frac{\pi}{4} = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{1}{\sqrt{2}} = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = -2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{1-\sqrt{2}}{\sqrt{2}} = 2\sin^2\left(\frac{\pi}{8}\right)$$

$$-\frac{1+\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{-\sqrt{2}+2}{2} = 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\therefore \sin^2\left(\frac{\pi}{8}\right) = \frac{2-\sqrt{2}}{4}$$

$$\sin \frac{\pi}{8} = \pm \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$\therefore \frac{\pi}{8} \text{ is in QI}$$

$$\therefore \sin \text{ is pos}$$

$$\therefore \sin \frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

- p.408 12. Use the appropriate compound angle formula and double angle formula to develop a formula for
- $\sin 3\theta$ in terms of $\cos \theta$ and $\sin \theta$
 - $\cos 3\theta$ in terms of $\cos \theta$ and $\sin \theta$
 - $\tan 3\theta$ in terms of $\tan \theta$

(a) $\sin 3\theta = \sin(2\theta + \theta)$

$$= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$$

$$= 2\sin \theta \cos \theta \cos \theta + (1 - 2\sin^2 \theta) \sin \theta$$

$$= 2\sin \theta \cos^2 \theta + \sin \theta - 2\sin^3 \theta$$

$$\text{or } = 3\cos^2 \theta \sin \theta - \sin^3 \theta$$

$$\text{or } = 4\cos^2 \theta \sin \theta - \sin \theta$$

(c) $\tan 3\theta$

$$= \tan(2\theta + \theta)$$

$$= \frac{\tan 2\theta + \tan \theta}{1 - \tan 2\theta \tan \theta}$$

$$= \frac{2\tan \theta + \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{1 - (\frac{2\tan \theta}{1 - \tan^2 \theta}) \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2\tan \theta}{1 - \tan^2 \theta} + \tan \theta \left(\frac{1 + \tan^2 \theta}{1 - \tan^2 \theta} \right)$$

$$= \frac{1 - \frac{2\tan^2 \theta}{1 - \tan^2 \theta}}{1 - \tan^2 \theta}$$

$$= \frac{2\tan \theta + \tan \theta - \tan^3 \theta}{1 - \tan^2 \theta}$$

$$= \frac{1 - \tan^2 \theta - 2\tan^2 \theta}{1 - \tan^2 \theta}$$

$$= \frac{3\tan \theta - \tan^3 \theta}{1 - \tan^2 \theta} \times \frac{1 - \tan^2 \theta}{1 - 3\tan^2 \theta}$$

$$= \frac{3\tan \theta - \tan^3 \theta}{1 - 3\tan^2 \theta}$$

(b) $\cos 3\theta = \cos(2\theta + \theta)$

$$= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$$

$$= (1 - 2\sin^2 \theta) \cos \theta - 2\sin \theta \cos \theta \sin \theta$$

$$= \cos \theta - 2\sin^2 \theta \cos \theta - 2\sin^2 \theta \cos \theta \sin \theta$$

$$= \cos \theta - 4\sin^2 \theta \cos \theta$$

$$\text{or } = 2\cos^3 \theta - \cos \theta - 2\sin^2 \theta \cos \theta$$

$$\text{or } = \cos^3 \theta - 3\sin^2 \theta \cos \theta$$

- p.408 16. Eliminate A from each pair of equations to find an equation that relates x to y .

- a) $x = \tan 2A, y = \tan A$ c) $x = \cos 2A, y = \csc A$
 b) $x = \cos 2A, y = \cos A$ d) $x = \sin 2A, y = \sec 4A$

$$\begin{aligned}
 & \text{(16a)} \quad x = \tan 2A, y = \tan A \quad \tan^{-1} x = 2A \quad \text{(16b)} \quad x = \cos 2A, y = \cos A \quad \cos^{-1} x = 2A \\
 & \left\{ \begin{array}{l} \tan 2A \\ = \frac{2 \tan A}{1 - \tan^2 A} \end{array} \right. \quad \begin{array}{l} \tan^{-1} y = A \\ \therefore \tan^{-1} y = \frac{\tan^{-1} x}{2} \end{array} \quad \left\{ \begin{array}{l} \cos 2A \\ = \frac{\cos^2 x - \cos^2 x}{2} \end{array} \right. \quad \begin{array}{l} \cos^{-1} y = A \\ \therefore \cos^{-1} y = \frac{\cos^{-1} x}{2} \end{array} \\
 & \text{or } \begin{array}{l} \therefore \tan^{-1} y = \frac{\tan^{-1} x}{2} \\ \therefore \cos^{-1} y = \frac{\cos^{-1} x}{2} \end{array} \quad \text{or } \begin{array}{l} \therefore \cos^{-1} y = \frac{\cos^{-1} x}{2} \\ \therefore \cos^{-1} y = \frac{\cos^{-1} x}{2} \end{array} \\
 & \text{(16c)} \quad x = \sin 2A \quad y = \sec 4A \quad \text{(16d)} \quad x = \sin 2A \quad y = \sec 4A \\
 & \left\{ \begin{array}{l} \sin^{-1} x = 2A \\ = \frac{\sin^{-1} x}{2} \end{array} \right. \quad \begin{array}{l} \sec^{-1} y = 4A \\ = \frac{\sec^{-1} y}{4} \end{array} \quad \left\{ \begin{array}{l} \sin^{-1} x = \frac{\sec^{-1} y}{2} \\ \text{or } \sin^{-1} x = \frac{\sec^{-1} y}{2} \end{array} \right. \quad \begin{array}{l} \sec^{-1} y = 4A \\ \therefore \sec^{-1} y = \frac{\sec^{-1} x}{4} \end{array} \\
 & \text{or } \begin{array}{l} \therefore \sin^{-1} x = \frac{\sec^{-1} y}{2} \\ \text{or } \sin^{-1} x = \frac{\sec^{-1} y}{2} \end{array} \quad \text{or } \begin{array}{l} \therefore \sin^{-1} x = \frac{\sec^{-1} y}{2} \\ \text{or } \sin^{-1} x = \frac{\sec^{-1} y}{2} \end{array}
 \end{aligned}$$

- p.408 17. Solve each equation for values of x in the interval $0 \leq x \leq 2\pi$.

- a) $\cos 2x = \sin x$ b) $\sin 2x - 1 = \cos 2x$

$$\begin{aligned}
 & \text{(17a)} \quad \cos 2x = \sin x \\
 & 1 - 2\sin^2 x = \sin x \\
 & 0 = 2\sin^2 x + \sin x - 1 \\
 & = (2\sin x - 1)(\sin x + 1) \\
 & \text{if } 2\sin x - 1 = 0 \quad \therefore \sin x = -1 \\
 & 2\sin x = 1 \quad x = \frac{3\pi}{2} \\
 & \sin x = \frac{1}{2} \\
 & \therefore x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6}
 \end{aligned}
 \quad
 \begin{aligned}
 & \text{(17b)} \quad \sin 2x - 1 = \cos 2x \\
 & 2\sin x \cos x - 1 = 1 - 2\sin^2 x \\
 & 2\sin^2 x + 2\sin x \cos x - 1 - 1 = 0 \\
 & 2\sin^2 x + 2\sin x \cos x - 2 = 0 \\
 & 2(\sin^2 x + \sin x \cos x - 1) = 0 \\
 & 2(\sin^2 x + \sin x \cos x - (\sin^2 x \cos^2 x)) = 0 \\
 & \sin^2 x + \sin x \cos x - \sin^2 x \cos^2 x = 0 \\
 & \sin x \cos x(1 - \cos^2 x) = 0 \\
 & \sin x \cos x(\sin^2 x) = 0
 \end{aligned}
 \quad
 \begin{aligned}
 & \therefore \cos x = 0 \text{ or } \sin x - \cos x = 0 \\
 & x = \frac{\pi}{2}, \frac{3\pi}{2} \\
 & \sin x = \cos x \\
 & \therefore x = \frac{\pi}{4} \\
 & \text{or } x = \frac{5\pi}{4}
 \end{aligned}$$

7.4 Proving Trigonometric Identities



"By the end of next class:

I can prove any identity using previously established identities.

To disprove a claim, I understand that I only require a counterexample to it.

I can apply what I have learned in unfamiliar settings."

Identities Based on Definitions

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Cofunction Identities

$$\sin \theta = \cos \left(\frac{\pi}{2} - \theta \right)$$

$$\cos \theta = \sin \left(\frac{\pi}{2} - \theta \right)$$

$$\tan \theta = \cot \left(\frac{\pi}{2} - \theta \right)$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Identities Derived from Relationships

Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Odd and Even Function Identities

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

Ex. 1: Prove $\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = \frac{2}{\sin^2\theta}$

Ex. 1

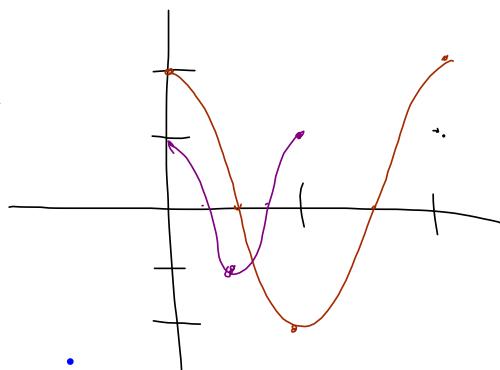
$$\begin{aligned} \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} &= \frac{2}{\sin^2\theta} \\ \text{LS} &= \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} \\ &= \frac{1-\cos\theta+1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)} \\ &= \frac{2}{1-\cos^2\theta} \quad (\cos^2\theta + \sin^2\theta = 1) \\ &= \frac{2}{\sin^2\theta} \quad \text{RS} = \frac{2}{\sin^2\theta} \\ \therefore \text{LS} &= \text{RS} \quad \therefore \text{QED}. \end{aligned}$$

Ex. 2: Show that $\cos 2x = 2\cos x$ is not an identity.

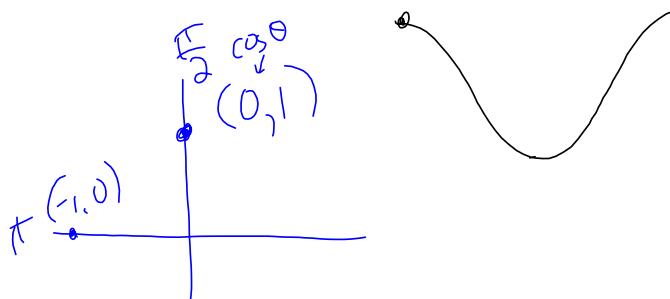
Let $x = \frac{\pi}{2}$

$$\begin{aligned} \text{LS} &= \cos 2x \\ &= \cos 2\left(\frac{\pi}{2}\right) \\ &= \cos \pi \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{RS} &= 2\cos x \\ &= 2\cos\left(\frac{\pi}{2}\right) \\ &= 2(0) \\ &= 0 \end{aligned}$$



$\therefore \text{LS} \neq \text{RS}$
 $\therefore \cos 2x \neq 2\cos x$
 IS NOT an identity.



Ex. 3: Use a compound angle identity to prove that

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

Ex 3. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$

$LS = \sin\left(\frac{\pi}{2} - \theta\right)$

$$= \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$$

$$= 1(\cos\theta) - 0(\sin\theta)$$

$$= \cos\theta$$

$$= RS$$

$RS = \cos\theta$

$\therefore LS = RS$

$\therefore \text{QED!}$

Today's Work: pp. 417-418 #1, 5ac, 8, 9abc, 17

Next class: p. 418 #10abce, 11bdgjl

The textbook answer section for 7.4 is poorly written.

For example, all identity proofs should always have a LS and RS chart; the answer section does not do this.