Last day's Work: p. 418 #10abce, 11bdgjl (?

10. Prove each identity.

b) $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

$LS = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + (\cos^2\theta)(\cos^2\theta)$

$= \sin^2\theta + (1-\sin^2\theta)(1-\sin^2\theta)$

$= \sin^2\theta + 1 - 2\sin^2\theta + \sin^4\theta$

$= \sin^4\theta + 1 - \sin^2\theta$

$= \sin^4\theta + \cos^2\theta$

$= RS$

$\therefore LS = RS$

$\therefore$ QED!

e) $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x$

$LS = \sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right)$

$= \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$

$= 2\sin\frac{\pi}{4}\cos x + 0$

$= 2\left(\frac{\sqrt{2}}{2}\right)\cos x$

$= \sqrt{2}\cos x$

$= RS$

$\therefore LS = RS$

$\therefore$ QED!

Last day's Work: p. 418 #10abce, 11bdgjl (?

11. Prove each identity.

b) $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$

$LS = \dfrac{\sin 2x}{1-\cos 2x}$

$= \dfrac{2\sin x\cos x}{1-(1-2\sin^2 x)}$

$= \dfrac{2\sin x\cos x}{1-1+2\sin^2 x}$

$= \dfrac{\cancel{2}\sin x\cos x}{\cancel{2}\sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

$= \cot x$

$= RS$

$\because LS = RS$

$\therefore$ QED.

i) $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

$LS = \dfrac{2\tan x}{1+\tan^2 x}$

$= \dfrac{2\tan x}{\sec^2 x}$

$= \dfrac{2\sin x}{\cos x} \div \sec^2 x$

$= \dfrac{2\sin x}{\cos x} \div \dfrac{1}{\cos^2 x}$

$= \dfrac{2\sin x}{\cos x} \times \dfrac{\cos^2 x}{1}$

$= 2\sin x\cos x$

$= \sin 2x$

$= RS$

$\because LS = RS$

$\therefore$ QED.

l) $\sec t = \dfrac{\sin 2t}{\sin t} - \dfrac{\cos 2t}{\cos t}$

$RS = \dfrac{\sin 2t}{\sin t} - \dfrac{\cos 2t}{\cos t}$

$= \dfrac{2\sin t\cos t}{\sin t} - \dfrac{2\cos^2 t - 1}{\cos t}$

$= 2\cos t - \left(\dfrac{2\cos^2 t}{\cos t} - \dfrac{1}{\cos t}\right)$

$= 2\cos t - 2\cos t + \dfrac{1}{\cos t}$

$= \dfrac{1}{\cos t}$

$= \sec t$

$= LS$

$\because RS = LS$

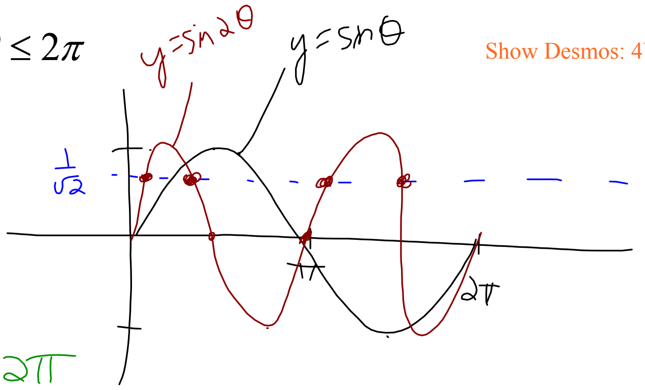
$\therefore$ QED.

7.5 Solving Linear Trigonometric Equations

"I can solve for the unknown angle(s) in any linear trigonometric equation.
I realize that I may need to apply previously established identities to do so.
I can apply what I have learned in unfamiliar settings."

Ex. 1: Solve: $\sin 2\theta = \frac{1}{\sqrt{2}}$, $0 \le \theta \le 2\pi$

Show Desmos: 4U 7.5 Ex.1



Let $\alpha = 2\theta$

$\sin\alpha = \frac{1}{\sqrt{2}}$

raa: $\sin\beta = \frac{1}{\sqrt{2}}$

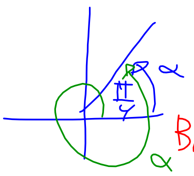
$\beta = \frac{\pi}{4}$

$0 \le \theta \le 2\pi$

$0 \le 2\theta \le 4\pi$

$0 \le \alpha \le 4\pi$

QI



$\alpha = \beta$

$= \frac{\pi}{4}$

But $\alpha = 2\theta$

$\therefore 2\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{8}$

or $\alpha = 2\pi + \beta$

$= 2\pi + \frac{\pi}{4}$

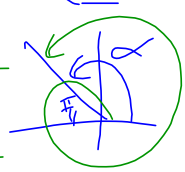
$= \frac{8\pi}{4} + \frac{\pi}{4}$

$= \frac{9\pi}{4}$

$2\theta = \frac{9\pi}{4}$

$\theta = \frac{9\pi}{8}$

QII



$\alpha = \pi - \beta$

$= \frac{3\pi}{4}$

$2\theta = \frac{3\pi}{4}$

$\theta = \frac{3\pi}{8}$

or $\alpha = 3\pi - \beta$

$= \frac{12\pi}{4} - \frac{\pi}{4}$

$= \frac{11\pi}{4}$

$2\theta = \frac{11\pi}{4}$

$\theta = \frac{11\pi}{8}$

$\therefore \{\theta \in \mathbb{R} \mid \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}\}$

Ex. 2: Solve over the interval given. State exact answers in radians.

$$\cos^2\theta - \sin^2\theta = \sqrt{3}\sin 2\theta,\ 0 \le \theta \le 2\pi$$

$$\cos 2\theta = \sqrt{3}\sin 2\theta$$

$$\frac{\cancel{\cos 2\theta}}{\cancel{\cos 2\theta}} = \frac{\sqrt{3}\sin 2\theta}{\cos 2\theta}$$

$$1 = \sqrt{3}\tan 2\theta$$

$$\frac{1}{\sqrt{3}} = \tan 2\theta$$

Let $A = 2\theta$

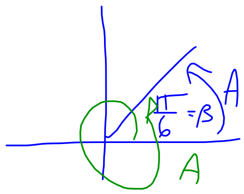
$$0 \le \theta \le 2\pi$$
$$0 \le 2\theta \le 4\pi$$
$$0 \le A \le 4\pi$$

$$\tan A = \frac{1}{\sqrt{3}}$$

raa $\rightarrow$ $\tan\beta = \frac{1}{\sqrt{3}}$

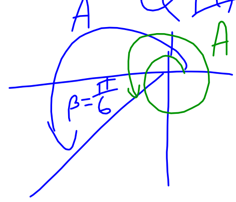
$$\beta = \frac{\pi}{6}$$

QI



2π $\quad \frac{12\pi}{6}$

QIII



$A = \frac{\pi}{6}$ or $A = 2\pi + \beta = \frac{13\pi}{6}$

$2\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{12}$

$2\theta = \frac{13\pi}{6}$

$\theta = \frac{13\pi}{12}$

$A = \pi + \beta = \frac{7\pi}{6}$ or $A = 3\pi + \frac{\pi}{6} = \frac{18\pi}{6} + \frac{\pi}{6} = \frac{19\pi}{6}$

$2\theta = \frac{7\pi}{6}$

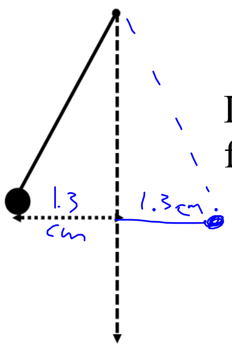
$\theta = \frac{7\pi}{12}$

$2\theta = \frac{19\pi}{6}$

$\theta = \frac{19\pi}{12}$

$$\therefore \left\{\theta \in \mathbb{R} \,\middle|\, \theta = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}\right\}$$

Ex. 3: A pendulum swings. The displacement from centre in centimetres (d) over time in seconds (t) is given by:



$$d = -2\cos\frac{\pi}{2}t$$

Determine the first two time values when the horizontal distance from centre is 1.3 cm, each rounded to the nearest hundredth.

$\pm 1.3 = -2\cos\frac{\pi}{2}t$

$-1.3 = -2\cos\frac{\pi}{2}t$ or $1.3 = -2\cos\frac{\pi}{2}t$

$\frac{-1.3}{-2} = \cos\frac{\pi}{2}t$ $\qquad$ $-0.65 = \cos\frac{\pi}{2}t$

$0.65 = \cos\frac{\pi}{2}t$

Let $A = \frac{\pi}{2}t$

$0.65 = \cos A$

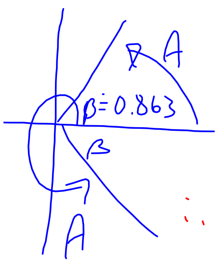
$A = \cos^{-1}(0.65)$

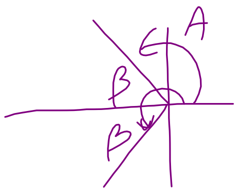
$\beta = \cos^{-1}(0.65)$

$\doteq 0.863$ radians

$\beta = \cos^{-1}(0.65)$

$\doteq 0.863$





$A = \beta$ or $A = 2\pi - \beta$

$\doteq 0.863$ $\qquad$ $\doteq 5.417$

$\therefore \frac{\pi}{2}t \doteq 0.863$ $\qquad$ $\frac{\pi}{2}t \doteq 5.417$

$t \doteq \frac{2}{\pi}(0.863)$ $\qquad$ $t \doteq 3.448$

$t \doteq 0.549$ $\qquad$ $t \doteq 3.45$

$\doteq 0.55$

$A \doteq \pi - \beta$ or $A = \pi + \beta$

$\doteq 2.278$ $\qquad$ $\doteq 4.003$

$\frac{\pi}{2}t \doteq 2.278$ $\qquad$ $\frac{\pi}{2}t \doteq 4.003$

$t \doteq 1.450$ $\qquad$ $t \doteq 2.548$

$\doteq 1.45$ $\qquad$ $\doteq 2.55$

the first two time values when the pendulum is 1.3 cm from centre are 0.55 sec and 1.45 sec.

Entertainment: pp.426-428 #3, 6de, 7de, 9de, 10ef, 11*, 13

*Hint: produce a sketch first