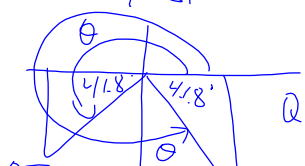
Last day's Work: pp.426-428 #3, 6de, 7de, 9de, 10ef, 11*, 13
*Hint: produce a sketch first

p.427

7. Using a calculator, determine the solutions for each equation on the interval $0° \le \theta \le 360°$. Express your answers to one decimal place.

a) $2 \sin \theta = -1$
b) $3 \cos \theta = -2$
c) $2 \tan \theta = 3$
d) $-3 \sin \theta - 1 = 1$
e) $-5 \cos \theta + 3 = 2$
f) $8 - \tan \theta = 10$

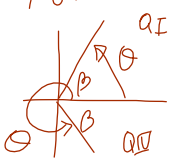
d) $-3\sin\theta - 1 = 1$
$-3\sin\theta = 1+1$
$-3\sin\theta = 2$
$\sin\theta = \frac{-2}{3}$
$\beta = \sin^{-1}\left(\frac{2}{3}\right)$
$\doteq 41.81°$



$\theta \doteq 180° + 41.8°$
$\doteq 221.8°$

$\theta \doteq 360° - 41.8°$
$\doteq 318.2°$

e) $-5\cos\theta + 3 = 2$
$-5\cos\theta = 2 - 3$
$-5\cos\theta = -1$
$\cos\theta = \frac{1}{5}$
$\theta = \cos^{-1}\left(\frac{1}{5}\right)$
$\doteq 78.46$



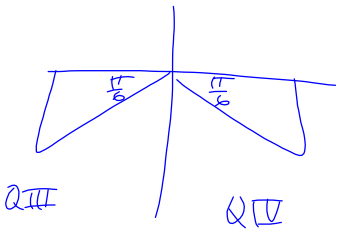
∴ QI
$\theta = \beta$
$\doteq 78.5°$

QIV:
$\theta = 360° - \beta$
$\doteq 281.5°$

9. Using a calculator, determine the solutions for each equation, to two decimal places, on the interval $0 \le x \le 2\pi$.

a) $2 - 2 \cot x = 0$
b) $\csc x - 2 = 0$
c) $7 \sec x = 7$
d) $2 \csc x + 17 = 15 + \csc x$
e) $2 \sec x + 1 = 6$
f) $8 + 4 \cot x = 10$

d) $2\csc x + 17 = 15 + \csc x$
$2\csc x - \csc x = 15 - 17$
$\csc x = -2$
$\frac{1}{\sin x} = \frac{-2}{1}$
$\sin x = \frac{-1}{2}$
$\beta = \frac{\pi}{6}$



$x = \pi + \beta$
$= \frac{7\pi}{6}$
$\doteq 3.67$

$x = 2\pi - \beta$
$= \frac{11\pi}{6}$
$\doteq 5.76$

p.427 **10.** Using a calculator, determine the solutions for each equation, to two decimal places, on the interval $0 \le x \le 2\pi$.

a) $\sin 2x = \frac{1}{\sqrt{2}}$ c) $\sin 3x = -\frac{\sqrt{3}}{2}$ e) $\cos 2x = -\frac{1}{2}$

b) $\sin 4x = \frac{1}{2}$ d) $\cos 4x = -\frac{1}{\sqrt{2}}$ f) $\cos \frac{x}{2} = \frac{\sqrt{3}}{2}$

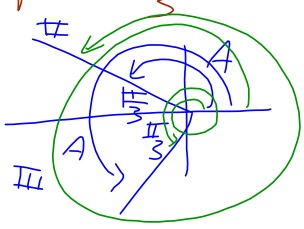
e) $\cos 2x = -\frac{1}{2}$

Let $A = 2x$

$\cos A = -\frac{1}{2}$ $0 \le x \le 2\pi$

$\beta = \frac{\pi}{3}$ $0 \le 2x \le 4\pi$

$0 \le A \le 4\pi$



QII: $A = \pi - \frac{\pi}{3}$ Q III

$= \frac{2\pi}{3}$ $A = \pi + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

$2x = \frac{2\pi}{3}$ $2x = \frac{4\pi}{3}$

$x = \frac{2\pi}{6}$ $x = \frac{2\pi}{3}$

$= \frac{\pi}{3}$

QII: Also Q IV

$A = 3\pi - \frac{\pi}{3}$ $A = 3\pi + \frac{\pi}{3}$

$= \frac{8\pi}{3}$ $= \frac{10\pi}{3}$

$2x = \frac{8\pi}{3}$ $2x = \frac{10\pi}{3}$

$x = \frac{4\pi}{3}$ $x = \frac{5\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

f) $\cos \frac{x}{2} = \frac{\sqrt{3}}{2}$

Let $w = \frac{x}{2}$

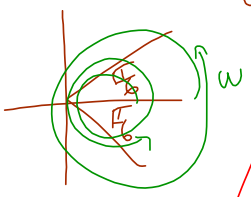
$\therefore \cos w = \frac{\sqrt{3}}{2}$

$\beta = \frac{\pi}{6}$

$Q_1: w = \frac{\pi}{6}$ Q IV

$w = 2\pi - \beta$

$= \frac{11\pi}{6}$



or

$w = 2\pi + \frac{\pi}{6}$ or

$= \frac{13\pi}{6}$ $w = 4\pi - \frac{\pi}{6}$

$= \frac{23\pi}{6}$

$\frac{x}{2} = \frac{13\pi}{6}$ $\frac{x}{2} = \frac{23\pi}{6}$

$w = \frac{\pi}{6}$ $x = \frac{13\pi}{3}$ $x = \frac{23\pi}{3}$

$2\left(\frac{x}{2}\right) = \left(\frac{\pi}{6}\right)2$

$x = \frac{\pi}{3}$

$\frac{x}{2} = \frac{11\pi}{6}$

$x = \frac{11\pi}{3}$

$2\pi = \frac{6\pi}{3}$

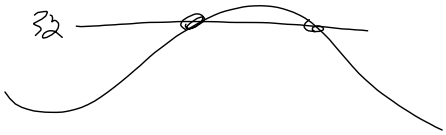
only $x = \frac{\pi}{3}$ is within $0 \le x \le 2\pi$.

p.428 **11.** A

A city's daily high temperature, in degrees Celsius, can be modelled by the function $t(d) = -28\cos\frac{2\pi}{365}d + 10$, where d is the day of the year and 1 = January 1. On days when the temperature is approximately 32 °C or above, the air conditioners at city hall are turned on. During what days of the year are the air conditioners running at city hall?

Find d, when $t(d)=32$

$32 = -28\cos\frac{2\pi}{365}d + 10$



$\frac{22}{-28} = \cos\frac{2\pi}{365}d$

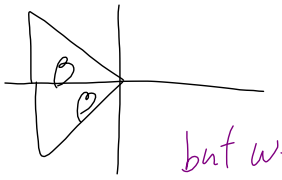
Let $w = \frac{2\pi}{365}d$

$\therefore \cos w = \frac{-22}{28}$

$w = \cos^{-1}\left(\frac{-11}{14}\right)$

$\beta = \cos^{-1}\left(\frac{11}{14}\right)$

$\beta = 0.6669$



$\therefore w = \pi - \beta$ or $w = \pi + \beta$

$= 2.474$ or $w = 3.808$

but $w = \frac{2\pi}{365}d$

$\frac{2\pi}{365}d = 2.474$

$d = \frac{365}{2\pi}(2.474)$

$= 143.75$ (May 23)

or

$d = \frac{365}{2\pi}(3.808)$

$= 221.2$ (Aug. 21)

13. Solve $\sin\left(x + \frac{\pi}{4}\right) = \sqrt{2}\cos x$ for $0 \le x \le 2\pi$.

$\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4} = \sqrt{2}\cos x$

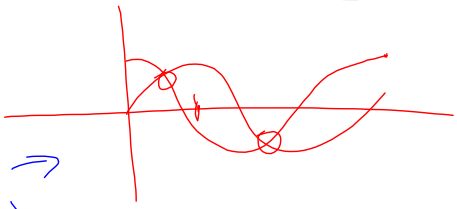
$\sin x\left(\frac{\sqrt{2}}{2}\right) + \cos x\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}\cos x$

$\frac{\sqrt{2}}{2}\sin x = \sqrt{2}\cos x - \frac{\sqrt{2}}{2}\cos x$

$= \frac{2\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\cos x$

$\frac{\sqrt{2}}{2}\sin x = \frac{\sqrt{2}}{2}\cos x$

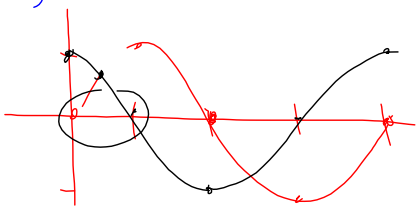
$\sin x = \cos x$ → Solve by graphing?



or

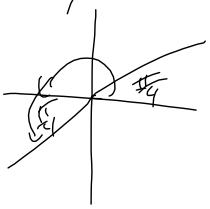
$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$



$x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$

(rad)



7.6 Solving Quadratic Trigonometric Equations

"I can solve for the unknown angle(s) in any quadratic trigonometric equation.
I realize that I may need to apply previously established identities to do so.
I can apply what I have learned in unfamiliar settings."

Ex. 1: Solve for x in the interval $0 \le x \le 2\pi$. (Round all final answers to the nearesthundredth.)

$$2\sec^2 x - 5\tan x = 5$$

$\tan^2 x + 1 = \sec^2 x$

$2(\tan^2 x + 1) - 5\tan x = 5$

$2\tan^2 x + 2 - 5\tan x - 5 = 0$

$2\tan^2 x - 5\tan x - 3 = 0$

Let $w = \tan x$

$2w^2 - 5w - 3 = 0$

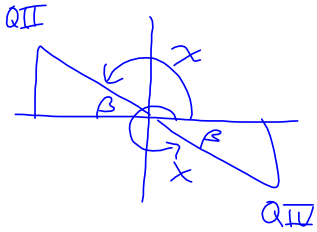
$(2w + 1)(w - 3) = 0$

$w = -\frac{1}{2}$ or $w = 3$

$\tan x = -\frac{1}{2}$

$\beta = \tan^{-1}\left(\frac{1}{2}\right)$

$\doteq 0.463$ radians ← raa



QII:

$x = \pi - \beta$

$\doteq 2.677$

$\doteq 2.68$

QIV:

$x = 2\pi - \beta$

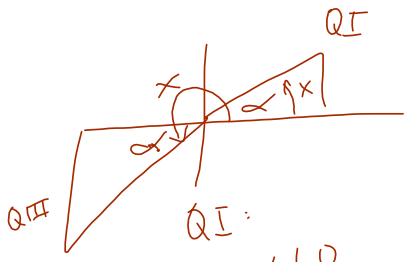
$\doteq 5.819$

$\doteq 5.82$

$\tan x = 3$

$\alpha = \tan^{-1}(3)$

$\doteq 1.249$ ← (raa)



QI:

$x = \alpha + 0$

$\doteq 1.249$

$\doteq 1.25$

QIII:

$x = \pi + \beta$

$\doteq 4.390$

$\doteq 4.39$

$\therefore x \doteq 1.25, 2.68, 4.39, 5.82$

Entertainment : pp. 436-437 #4d*, 5c*, 6b*c*, 7a*, 7e, 8d*, 8f*, 9d *, 14, 17
* means no rounding!!!!

Ex. 2 Solve for θ on $0 \le \theta \le 2\pi$

$$3\tan^2\theta = 1$$

$$\tan^2\theta = \frac{1}{3}$$

$$\tan\theta = \pm\sqrt{\frac{1}{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \text{or} \quad \tan\theta = -\frac{1}{\sqrt{3}}$$

$$\text{ra} = \beta = \frac{\pi}{6} \longrightarrow$$

$$\therefore \theta = \frac{\pi}{6} \text{ or } \theta = \frac{7\pi}{6} \qquad \therefore \theta = \frac{5\pi}{6} \text{ or } \theta = \frac{11\pi}{6}$$