Last day's Work: p. 440
#2, 3d, 4a, 5ab, 6a, 8*, 9*, 10b, 12abc** DO 12 c LAST.

*as usual, set up LS/RS separately

**for 12c, although using the quadratic formula yields the solution, the final simplification process is difficult. Instead, expand the LS, then "factor by grouping."

p. 440 **4.** Evaluate each expression.

a) $\dfrac{\tan\frac{\pi}{12} + \tan\frac{7\pi}{4}}{1 - \tan\frac{\pi}{12}\tan\frac{7\pi}{4}}$

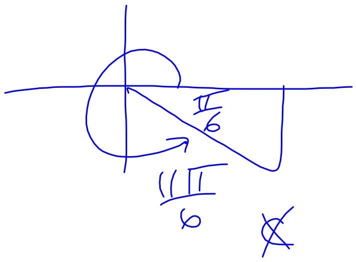
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \tan\left(\frac{\pi}{12} + \frac{7\pi}{4}\right)$$

$$= \tan\left(\frac{\pi}{12} + \frac{21\pi}{12}\right)$$

$$= \tan\left(\frac{22\pi}{12}\right)$$

$$= \tan\left(\frac{11\pi}{6}\right)$$



$$= -\tan\left(\frac{\pi}{6}\right)$$

$$= -\frac{\sqrt{3}}{3}$$

9. Prove that $\dfrac{2\sec^2 x - 2\tan^2 x}{\csc x} = \sin 2x \sec x$ is a trigonometric identity.

$$LS = \frac{2\sec^2 x - 2\tan^2 x}{\csc x}$$

$$= \frac{2(\sec^2 x - \tan^2 x)}{\csc x}$$

$$= \frac{2(1+\tan^2 x - \tan^2 x)}{\csc x}$$

$$= \frac{2(1)}{\csc x}$$

$$= 2\left(\frac{1}{\csc x}\right)$$

$$= 2(\sin x)$$

$$= 2\sin x$$

$$RS = \sin 2x \sec x$$

$$= 2\sin x \cos x \sec x$$

$$= 2\sin x \cancel{\cos x}\left(\frac{1}{\cancel{\cos x}}\right)$$

$$= 2\sin x$$

$\therefore LS = RS$

$\therefore$ QED

This is an alternate solution to 4a)
It works, but as you read it you will see that it is not as good a choice.

p. 440 **4.** Evaluate each expression.

a) $\dfrac{\tan\frac{\pi}{12} + \tan\frac{7\pi}{4}}{1 - \tan\frac{\pi}{12}\tan\frac{7\pi}{4}}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$\frac{\pi}{6}$ $\frac{2\pi}{12}$
$\frac{\pi}{4}$ $\frac{3\pi}{12}$
$\frac{\pi}{3}$ $\frac{4\pi}{12}$
$\frac{\pi}{2}$ $\frac{6\pi}{12}$

$$\frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$$

$\tan\frac{\pi}{4} = 1$
$\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$
$\tan\frac{7\pi}{4} = -1$

$\left(\tan\frac{7\pi}{4}\right.$

$\therefore \tan\frac{7\pi}{4} = -\tan\frac{\pi}{4} = -1$

$$= \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) + \tan\frac{7\pi}{4}}{1 - \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\tan\frac{7\pi}{4}}$$

$$= \frac{\dfrac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}} + \tan\frac{7\pi}{4}}{1 - \left(\dfrac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right)\tan\frac{7\pi}{4}}$$

$$= \frac{\dfrac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} + (-1)}{1 - \left(\dfrac{1 - \frac{\sqrt{3}}{3}}{1 + 1\left(\frac{\sqrt{3}}{3}\right)}\right)(-1)}$$

$$= \frac{\left(\frac{3-\sqrt{3}}{3}\right) \div \left(\frac{3+\sqrt{3}}{3}\right) - 1}{1 - \left[\left(\frac{3-\sqrt{3}}{3}\right) \div \left(\frac{3+\sqrt{3}}{3}\right)\right](-1)}$$

$$= \frac{\frac{3-\sqrt{3}}{\cancel{3}} \times \frac{\cancel{3}}{3+\sqrt{3}} - 1}{1 - \left(\frac{3-\sqrt{3}}{\cancel{3}} \times \frac{\cancel{3}}{3+\sqrt{3}}\right)(-1)}$$

$$= \frac{\frac{3-\sqrt{3}}{3+\sqrt{3}} - 1}{1 - (-1)\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right)}$$

$$= \frac{\dfrac{(3-\sqrt{3}) - (3+\sqrt{3})}{3+\sqrt{3}}}{\dfrac{(3+\sqrt{3}) + (3-\sqrt{3})}{3+\sqrt{3}}}$$

$$= \frac{3-\sqrt{3}-3-\sqrt{3}}{\cancel{3+\sqrt{3}}} \times \frac{\cancel{3+\sqrt{3}}}{3+\cancel{\sqrt{3}}+3-\cancel{\sqrt{3}}}$$

$$= \frac{-2\sqrt{3}}{6}$$

$$= \frac{-\sqrt{3}}{3}$$

$\therefore$ Previous screen solution was a much better choice!

p. 440 **12.** Solve each equation for x in the interval $0 \le x \le 2\pi$.

a) $2\sin^2 x - \sin x - 1 = 0$

b) $\tan^2 x \sin x - \dfrac{\sin x}{3} = 0$

c) $\cos^2 x + \left(\dfrac{1-\sqrt{2}}{2}\right)\cos x - \dfrac{\sqrt{2}}{4} = 0$

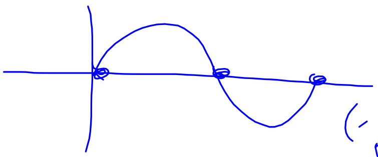
d) $25\tan^2 x - 70\tan x = -49$

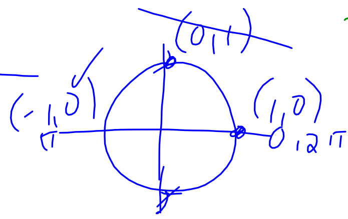
b) $\tan^2 x \sin x - \dfrac{\sin x}{3} = 0$

$3\tan^2 x \sin x - \sin x = 0$

$\sin x(3\tan^2 x - 1) = 0$

$\sin x = 0$ or $3\tan^2 x - 1 = 0$





$x = 0, \pi, 2\pi$

$\tan^2 x = \frac{1}{3}$

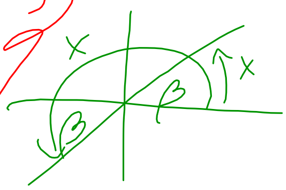
$\tan x = \pm\sqrt{\frac{1}{3}}$

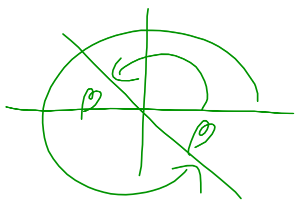
$= \frac{\pm 1}{\sqrt{3}}$

$\tan x = \frac{1}{\sqrt{3}}$ or $\tan x = \frac{-1}{\sqrt{3}}$

$\beta = \frac{\pi}{6}$ $\longrightarrow$

$\{x \in \mathbb{R} \mid x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi\}$





$x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$

$x = \frac{5\pi}{6}$ $x = \frac{11\pi}{6}$

p. 440 **12.** Solve each equation for x in the interval $0 \leq x \leq 2\pi$.

a) $2\sin^2 x - \sin x - 1 = 0$

b) $\tan^2 x \sin x - \dfrac{\sin x}{3} = 0$

c) $\cos^2 x + \left(\dfrac{1-\sqrt{2}}{2}\right)\cos x - \dfrac{\sqrt{2}}{4} = 0$

d) $25\tan^2 x - 70\tan x = -49$

$y^2 + \_\_y - \_\_ = 0$

2c)

$$\cos^2 x + \frac{1-\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{4} = 0$$

$$4(\cos^2 x) + \cancel{4}^{2}\left(\frac{1-\sqrt{2}}{\cancel{2}}\cos x\right) - \cancel{4}\left(\frac{\sqrt{2}}{\cancel{4}}\right) = 4(0)$$

$$4\cos^2 x + 2\cos x - 2\sqrt{2}\cos x - \sqrt{2} = 0$$

$$2\cos x(2\cos x + 1) - \sqrt{2}(2\cos x + 1) = 0$$

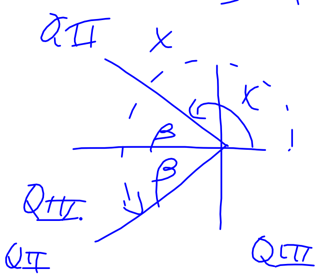
$$(2\cos x + 1)(2\cos x - \sqrt{2}) = 0$$

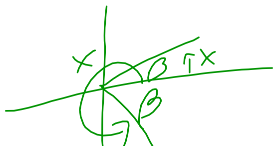
$2\cos x + 1 = 0$ or $2\cos x - \sqrt{2} = 0$

$\cos x = \frac{-1}{2}$ $\qquad$ $\cos x = \frac{\sqrt{2}}{2}$

$\beta = \cos^{-1}\left(\frac{1}{2}\right)$ $\qquad$ $\beta = \frac{\pi}{4}$

$\beta = \frac{\pi}{3}$ (raa)





$\therefore x = \frac{\pi}{4}$ or $x = 2\pi - \beta$
$= \frac{7\pi}{4}$

QII $\qquad$ QIII

$x = \pi - \beta$ $\qquad$ $x = \pi + \beta$

$= \frac{2\pi}{3}$ $\qquad$ $= \frac{4\pi}{3}$

$$\therefore \left\{x \in \mathbb{R} \,\middle|\, x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{4}\right\}$$