7.R2-2 SelfTest Corrections (Fall 2018)-f18p   December 14, 2018

Last day's Work: Chapter Self-Test p. 441 #1-10

9c, 5, 4, 8

p. 441 **4.** The quadratic trigonometric equation $a\cos^2 x + b\cos x - 1 = 0$ has the solutions $\frac{\pi}{3}$, π, and $\frac{5\pi}{3}$ in the interval $0 \le x \le 2\pi$. What are the values of a and b?

$a\cos^2\left(\frac{\pi}{3}\right)+b\cos\left(\frac{\pi}{3}\right)-1=0$

$a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)-1=0$

$\frac{a}{4}+\frac{b}{2}-1=0$

$\times 4$

$a+2b-4=0$

$2\cos^2 x+\cos x-1=0$

$a\cos^2(\pi)+b\cos(\pi)-1=0$

$a(-1)^2+b(-1)-1=0$

$a-b-1=0$

$-a-2b+4=0$

$-3b+3=0$

$b=1$

$\therefore a-(1)-1=0$

$a-2=0$

$a=2$

p. 441 **5.** The depth of the ocean at a swim buoy can be modelled by the function $d(t) = 4 + 2\sin\left(\frac{\pi}{6}t\right)$, where d is the depth of water in metres and t is the time in hours, if $0 \le t \le 24$. Consider a day when $t = 0$ represents midnight. Determine when the depth of water is 3 m.

$0\le t\le 24$

$0\le \frac{\pi}{6}t\le 4\pi \leftarrow \frac{\pi}{6}(24) = 4\pi$

$0\le A\le 4\pi$

Find t, when $d(t)=3$

$3=4+2\sin\left(\frac{\pi}{6}t\right)$

$3-4=2\sin\left(\frac{\pi}{6}t\right)$

$-\frac{1}{2}=\sin\left(\frac{\pi}{6}t\right)$

Let $A=\frac{\pi}{6}t$

$\sin A=-\frac{1}{2}$

$\beta=\sin^{-1}\left(\frac{1}{2}\right)$

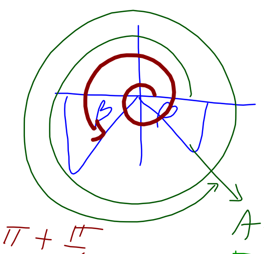
$\beta=\frac{\pi}{6}$

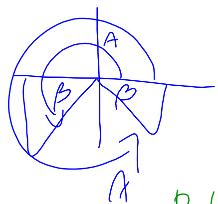
$\therefore A=\pi+\frac{\pi}{6}$

$=\frac{7\pi}{6}$

or $A=2\pi-\frac{\pi}{6}$

$=\frac{11\pi}{6}$





$A=3\pi+\frac{\pi}{6}$

$\frac{\pi}{6}t=\frac{19\pi}{6}$

$\therefore t=19$

(1900 = 7pm)

$A=4\pi-\frac{\pi}{6}$

$\frac{\pi}{6}t=\frac{23\pi}{6}$

$t=23$

$\therefore$ 2300 hr = 11 pm

But $A=\frac{\pi}{6}t$

$\therefore \frac{\pi}{6}t=\frac{7\pi}{6}$ $\qquad \frac{\pi}{6}t=\frac{11\pi}{6}$

$\therefore t=7$ $\qquad t=11$

$\therefore$ the depth of the water is 3 m at 7am, 11am, 7pm + 11 pm.

p. 441 **8.** The tangent of the acute angle α is 0.75, and the tangent of the acute angle β is 2.4. Without using a calculator, determine the value of $\sin(\alpha - \beta)$ and $\cos(\alpha + \beta)$. SYR CXR TYX

$\tan\alpha = 0.75$

$= \frac{3}{4}$

$\therefore y = 3,\ x = 4$

$\therefore r^2 = x^2 + y^2$

$r = 5$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\tan\beta = 2.4$

$= \frac{24}{10}$

$= \frac{12}{5}$

$\therefore y = 12,\ x = 5$

$\therefore r = 13$

$\sin\beta = \frac{12}{13}$

$\cos\beta = \frac{5}{13}$

$\sin(\alpha - \beta)$

$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{15}{65} - \frac{48}{65}$

$= \frac{-33}{65}$

p. 441 **9.** The angle x lies in the interval $\frac{\pi}{2} \leq x \leq \pi$, and $\sin^2 x = \frac{4}{9}$. Determine the value of each of the following. Round your answers to four decimal places.

a) $\sin 2x$ c) $\cos \frac{x}{2}$

b) $\cos 2x$ d) $\sin 3x$

Aside:

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\sin 2x = 2\sin x \cos x$$
$$\sin 10x = 2\sin 5x \cos 5x$$
$$\sin 3x = 2\sin \tfrac{3}{2}x \cos \tfrac{3}{2}x$$
$$\text{or } \sin 3x = \sin(2x + x)$$
$$=$$

Needed for all parts of #9:

$$\sin^2\theta = \frac{4}{9} \quad \text{in QII}$$

$$\sin\theta = \pm\sqrt{\frac{4}{9}}$$
$$\sin\theta = \frac{2}{3} \text{ or } \cancel{\sin x = \frac{-2}{3}}$$
$$\therefore y = 2,\ r = 3 \text{ or } \cancel{y = -2}$$
$$\therefore x^2 = r^2 - y^2$$
$$= 3^2 - 2^2$$
$$= 5$$
$$x = \pm\sqrt{5} \quad \text{But in QII } x = -\sqrt{5}$$

$$\therefore \sin\theta = \frac{2}{3} \text{ and } \cos\theta = \frac{-\sqrt{5}}{3}$$

c) $$\cos\frac{\theta}{2} = 1 - 2\sin^2\frac{\theta}{4}$$

But let's use a different approach:

Also

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\therefore \cos\theta = 2\cos^2\frac{\theta}{2} - 1$$
$$\cos\theta + 1 = 2\cos^2\frac{\theta}{2}$$
$$\frac{\cos\theta + 1}{2} = \cos^2\frac{\theta}{2}$$
$$\pm\sqrt{\frac{\cos\theta + 1}{2}} = \cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{\cos\theta + 1}{2}}$$

$\cos\theta = \frac{-\sqrt{5}}{3}$ and $\cos\frac{\theta}{2}$ is positive in QII:

$$\cos\frac{\theta}{2} = +\sqrt{\frac{\frac{-\sqrt{5}}{3} + \frac{3}{3}}{2}}$$
$$= \sqrt{\left(\frac{3 - \sqrt{5}}{3}\right) \div 2}$$
$$= \sqrt{\frac{3 - \sqrt{5}}{6}}$$

p. 441 **9.** The angle x lies in the interval $\frac{\pi}{2} \leq x \leq \pi$, and $\sin^2 x = \frac{4}{9}$. Determine the value of each of the following. Round your answers to four decimal places.

a) $\sin 2x$ c) $\cos \frac{x}{2}$

b) $\cos 2x$ d) $\sin 3x$

From previous page: in Q II

$\therefore \sin\theta = \frac{2}{3}$ and $\cos\theta = \frac{-\sqrt{5}}{3}$

Aside:

$\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (1-2\sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + (1\sin\theta - 2\sin^3\theta)$

$= 2\sin\theta(1-\sin^2\theta) + 1\sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + 1\sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$= 3\left(\frac{2}{3}\right) - 4\left(\frac{2}{3}\right)^3$$

$$= 2 - 4\left(\frac{8}{27}\right)$$

$$= \frac{54}{27} - \frac{32}{27}$$

$$= \frac{22}{27}$$