Before we begin, are there any questions from last day's work?

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) calculate the arc length of a circle.
b) calculate the area of a sector of a circle.

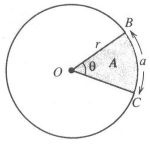
Cake Decorating assignment due today.

Project Renovation assignment past due.

6.9.1: Arc Length and Sector Area

Date: Dec. 17/18

There are problems that often occur in industry that involve arcs and sectors of circles.
Consider the following diagram:



A circle of radius r is drawn, with sector *BOC* bounded by 2 radii, *OB* and *OC*, and an arc *BC*, of length *a*.
The area of the sector is A,
and the sector angle at the centre *O* is θ, measured in degrees.

We can use the proportional relationship: $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{sector area}}{\text{area of circle}} = \frac{\text{sector angle}}{\text{complete rotation}}$

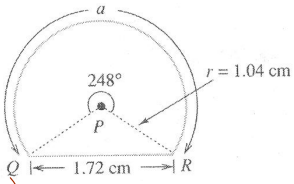
So, if $\frac{a}{2\pi r} = \frac{A}{\pi r^2} = \frac{\theta}{360°}$

then $\frac{a}{2\pi r} = \frac{\theta}{360°}$ and $\frac{A}{\pi r^2} = \frac{\theta}{360°}$

and isolating, [*arc length*] $a = 2\pi r\left(\frac{\theta}{360°}\right)$ and [*sector area*] $A = \pi r^2\left(\frac{\theta}{360°}\right)$

Thus both the arc length, *a*, and sector area, *A*, can be calculated
once the radius, *r*, and the sector angle, θ, in degrees, are known.

Ex. 1 A cam for a sewing machine's stitching-control cycle is circular in shape, with a flat side, and has the dimensions shown.
a) Calculate the total perimeter of the cam. (to 3 decimal places)



$P = \_\_ + \frown$

$= 1.72 + a$

$\doteq 1.72 + 4.502$

$\doteq 6.222$ cm

$a = \frac{\theta}{360}(2\pi r)$

$= \frac{248°}{360°}(2\pi(1.04))$

$\doteq 4.5015$

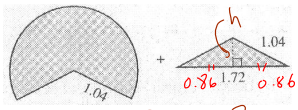
$\doteq 4.502$

∴ the total perimeter of the cam is 6.222 cm

b) If the cam is 0.36 cm thick and is made from an alloy whose density is 3.8g/cm^3, determine the mass of the cam. (to 3 decimal places)
Hint: the cam is a prism, Volume $_{\text{prism}}$ = Area $_{\text{base}}$ x height, then mass = volume x density

$M = DV$

$1.04^2 \doteq h^2 +$



$A_{\underset{\text{base}}{\text{cam}}} = A_{\text{prism}}$

$= A_{\text{Sector}} + A_{\text{Triangle}}$

$= \frac{\theta}{360}(\pi r^2) + \frac{1}{2}bh$

$\doteq \frac{248°}{360°}(\pi(1.04)^2) + \frac{1}{2}(1.72)(0.585)$

$\doteq 2.3408 + 0.5031$

$\doteq 2.8439$

$\doteq 2.844$ cm^2

$h^2 = 1.04^2 - 0.86^2$

$\doteq \sqrt{0.342}$

$\doteq 0.5848$

$\doteq 0.585$

$\text{Volume}_{\text{cam}} = A_{\text{base}} \times h$

$\doteq 2.844 \times 0.36$

$\doteq 1.0238$

$\doteq 1.024$ cm^3

$M = DV$

$\doteq 3.8\frac{g}{cm^3} \times 1.024 cm^3$

$\doteq 3.8912$

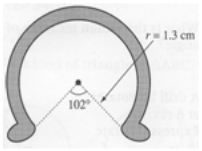
$\doteq 3.891$ g

∴ the mass of the cam is 3.891 g.

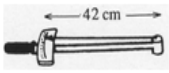
6.9.2 Arc Length and Sector Area

Date: _____

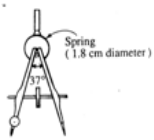
1. A snap-ring retainer clip with dimensions shown is part of a universal joint assembly, and fits snugly around the bearing cap when assembled. What length of the clip is in contact with the bearing cap? (to 2 decimal places)



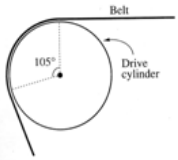
2. The pointer on a torque wrench is 42 cm long, and moves through an angle of 16°. Through what distance does the tip of the pointer move? (to 1 decimal place)



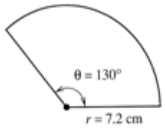
3. Calculate the length of the spring on the bow compasses shown. (to 2 decimal places)



4. The belt on a copier machine is in contact with a drive cylinder over 105° of its surface, as shown. If the length of contact is 5.62 cm, what is the radius of the drive cylinder? (to 2 decimal places)



5. The vent cover on a forced air heating system is in the form of a sector of a circle. Determine the area of sheet metal used to make the vent cover. (to 1 decimal place)



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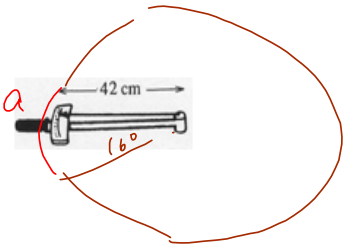
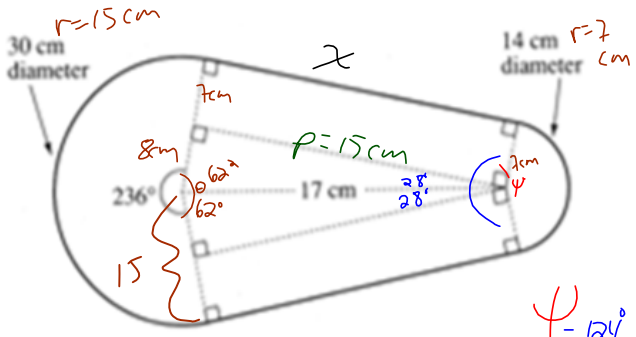


$a = \frac{\theta}{360}(2\pi r)$

$= \frac{16^\circ}{360^\circ}(2\pi(42))$

$=$

6. The safety shield for a motor pulley drive has the dimensions shown.
   a) Calculate the perimeter of the shield. (to 1 decimal place)
   b) Determine the area of the shield. (to 1 decimal place)



$P = (+ \text{—} +) + /$

$= (+ \text{—} +) + \text{—}$

$= \frac{236^\circ}{360^\circ}(2\pi(15))$

$\doteq 61.7846$

$\doteq 61.785$ cm

$a = \frac{124^\circ}{360^\circ} 2\pi(7)$

$\doteq 15.1494$

$\doteq 15.149$ cm

$\therefore P \doteq 61.785 + 15 + 15.149 + 15$

$\doteq 106.93$

$\doteq 106.9$ cm

$\theta = 360^\circ - 236^\circ$

$= 124^\circ$

$\therefore \frac{\theta}{2} = 62^\circ$

$\psi = 124^\circ$

$P^2 = 17^2 - 8^2$

$= 289 - 64$

$= 225$

$\therefore P = 15$

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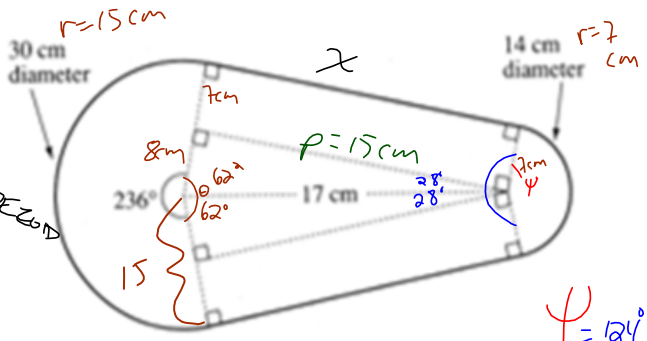


b) $A_{total} = A_{sector1} + A_{sector2} + 2A_{TRAPEZOID}$

$\doteq 846.40$

$\doteq 846.4 \text{ cm}^2$

$A_{sector1} = \frac{\theta}{360} \pi r^2$

$= \frac{236^\circ}{360^\circ} \pi (15)^2$

$\doteq 463.38$

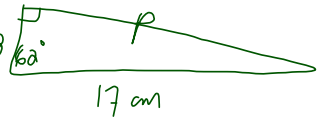
$A_{sector2} = \frac{124^\circ}{360^\circ} \pi (7)^2$

$\doteq 53.02$

$\theta = 360^\circ - 236^\circ$

$= 124^\circ$

$\therefore \frac{\theta}{2} = 62^\circ$



$p^2 = 17^2 - 8^2$

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$= 225$

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$A_{TRAP} = \frac{1}{2} h(a+b)$

$= \frac{1}{2}(15)(15+7)$

$= 165$

$\therefore 2 \text{ TRAPEZOIDS} = 330 \text{ cm}^2$

A_{Shield}