**Last day's Work:** pp.485-486 #1ce, 2ce, 3ce, 4ad, 5cde, 7, 8bf, 10, 11, 12

p.485 **4.** The formula to calculate the mass, $M(t)$, remaining from an original sample of radioactive material with mass $P$, is determined using the formula $M(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where $t$ is time and $h$ is the half-life of the substance. The half-life of a radioactive substance is 8 h. How long will it take for a 300 g sample to decay to each mass?

a) 200 g  b) 100 g  c) 75 g  d) 20 g

4. $M(t) = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$

a) $M(t) = 200$ g

$$200 = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$$

$$\frac{2}{3} = \frac{1}{2}^{\frac{t}{8}}$$

$$\log\left(\frac{2}{3}\right) = \frac{t}{8}\log\left(\frac{1}{2}\right)$$

$$\frac{8\log\left(\frac{2}{3}\right)}{\log\left(\frac{1}{2}\right)} = t$$

$$t \doteq 4.679$$

$$\doteq 4.68 \text{ h}$$

d) $M(t) = 20$ g

$$20 = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$$

$$\frac{2}{30} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$$

$$\log\left(\frac{2}{30}\right) = \frac{t}{8}\log\left(\frac{1}{2}\right)$$

$$\frac{8\log\left(\frac{2}{30}\right)}{\log\left(\frac{1}{2}\right)} = t$$

$$t \doteq 31.255$$

$$\doteq 31.26 \text{ h}$$

p.485 **7.** A bacteria culture doubles every 15 min. How long will it take for a culture of 20 bacteria to grow to a population of 163 840?

7. $P = P_0(2)^{\frac{t}{15}}$

$$163\,840 = 20(2)^{\frac{t}{15}}$$

$$\frac{163\,840}{20} = 2^{\frac{t}{15}}$$

$$\log 8192 = \frac{t}{15}\log 2$$

$$\frac{15\log 8192}{\log 2} = t$$

$$t = 195$$

OR

$$8192 = 2^{\frac{t}{15}}$$

$$2^{13} = 2^{\frac{t}{15}}$$

$$\therefore t = 13 \times 15$$

$$= 195$$

**Last day's Work:** pp.485-486 #1ce, 2ce, 3ce, 4ad, 5cde, 7, 8bf, 10, 11, 12

c

p.486 **8.** Solve for $x$.

a) $4^{x+1} + 4^x = 160$
b) $2^{x+2} + 2^x = 320$
c) $2^{x+2} - 2^x = 96$
d) $10^{x+1} - 10^x = 9000$
e) $3^{x+2} + 3^x = 30$
f) $4^{x+3} - 4^x = 63$

8b) $2^{x+2} + 2^x = 320$

$2^x(2^2+1) = 320$

$2^x(5) = 320$

$2^x = \frac{320}{5}$

$= 64$

$\therefore x = 6$

p.486 **10.** Solve. Round your answers to three decimal places.

a) $5^{t-1} = 3.92$
b) $x = \log_3 25$
c) $4^{2x} = 5^{2x-1}$
d) $x = \log_2 53.2$

c) $4^{2x} = 5^{2x-1}$

$2x\log 4 = (2x-1)\log 5$

$2x\log 4 = 2x\log 5 - \log 5$

$2x\log 4 - 2x\log 5 = -\log 5$

$x(2\log 4 - 2\log 5) = -\log 5$

$x = \frac{-\log 5}{2\log 4 - 2\log 5}$

$\doteq 3.6062 \longrightarrow \doteq 3.606$

**Last day's Work:** pp.485-486 #1ce, 2ce, 3ce, 4ad, 5cde, 7, 8bf, 10, 11, 12

p.486 **11.** A plastic sun visor allows light to pass through, but reduces the intensity of the light. The intensity is reduced by 5% if the plastic is 1 mm thick. Each additional millimetre of thickness reduces the intensity by another 5%.

a) Use an equation to model the relation between the thickness of the plastic and the intensity of the light.

b) How thick is a piece of plastic that reduces the intensity of the light to 60%?

11a) $b = 100\% - 5\%$

$= 95\%$

$= 0.95$

$\therefore I = I_0(0.95)^t$

11b) $I = 60, I_0 = 100$

$60 = 100(0.95)^t$

$0.6 = 0.95^t$

$\log 0.6 = t \log 0.95$

$t = \frac{\log 0.6}{\log 0.95}$

$\doteq 9.95$

$\doteq 10$ mm thick

p.486 **12.** Solve $3^{2x} - 5(3^x) = -6$.

$3^{2x} = (3^2)^x$ or $(3^x)^2$

$= (3^x)(3^x)$

$3^{2x} - 5(3^x) + 6 = 0$

Let $A = 3^x$

$A^2 - 5A + 6 = 0$

$(A-3)(A-2) = 0$

$(3^x - 3)(3^x - 2) = 0$

$3^x = 3$ or $3^x = 2$

$x = 1$

$\log 3^x = \log 2$

$x \log 3 = \log 2$

$x = \frac{\log 2}{\log 3}$

$\doteq 0.630$

$\doteq 0.63$

# 8.5 Solving Exponential Equations: Applications

"I can solve and check any equation that contains exponential expressions. I can apply what I have learned in unfamiliar settings."

**Recall:**

$f(x) = ab^x$ ← the number of growth/decay periods

final amount (or number)

initial amount (or number)

↳ b is the multiplier

$[b = 1 \pm r \leftarrow$ rate as a decimal$]$

$b = 1 + r \rightarrow$ exponential growth

$b = 1 - r \rightarrow$ exponential decay

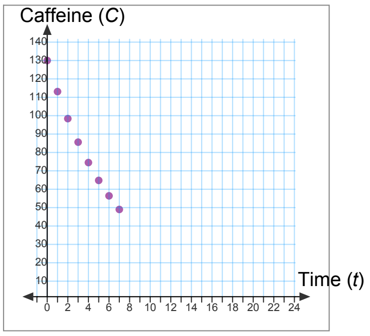
example: 2%

i) 2% growth
↳ 100% + 2%
= 102%
= 1.02

ii) 2% decay
↳ 100% − 2%
= 98%
= 0.98

Caffeine is present in most coffees, teas, chocolates and several other food and beverage products. Research shows that this chemical is eliminated from the human body over time exponentially (mostly). We will verify this using observed data.

The table and graph show the mass of caffeine ($C$) remaining in an average-sized person, for 7 hours after drinking a cup of coffee that contained 130 mg of caffeine.




| $t$ (hours) | $C$ (mg) |
|---|---|
| 0 | 130 |
| 1 | 113.1 |
| 2 | 98.4 |
| 3 | 85.6 |
| 4 | 74.5 |
| 5 | 64.8 |
| 6 | 56.4 |
| 7 | 49 |

y-ratios

$\frac{113.1}{130} = 0.87$

$\frac{98.4}{113.1} \doteq 0.8700...$

$\frac{56.4}{64.8} \doteq 0.8703...$

$\therefore b \doteq 0.87$

(comparisons on next page)

If $C$ represents mass of caffeine remaining, in mg, and $t$ represents the time, in hours, after drinking the coffee, calculate the half-life of caffeine, to the nearest hundredth.

$f(x) = ab^x$

$C(t) = ab^t$

$C(t) \doteq 130(0.87)^t$

To find the half-life $C(t) = \frac{130}{2}$
$= 65$

$\therefore 65 = 130(0.87)^t$

$\frac{65}{130} \doteq 0.87^t$

$0.5 \doteq 0.87^t$

$\log 0.5 \doteq \log 0.87^t$

$\log 0.5 \doteq t \log 0.87$

$\frac{\log 0.5}{\log 0.87} \doteq t$

$t \doteq 4.977$

$\doteq 4.98$ hours

Yesterday: $M(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$

or $C(t) = 130\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$C(6) = 56.4$

$56.4 \doteq 130\left(\frac{1}{2}\right)^{\frac{6}{h}}$

$\frac{56.4}{130} \doteq \left(\frac{1}{2}\right)^{\frac{6}{h}}$

$\log \frac{56.4}{130} \doteq \log\left(\frac{1}{2}\right)^{\frac{6}{h}}$

$\log \frac{56.4}{130} \doteq \frac{6}{h} \log \frac{1}{2}$

$h \log \frac{56.4}{130} \doteq 6 \log \frac{1}{2}$

$h \doteq \frac{6 \log \frac{1}{2}}{\log \frac{56.4}{130}}$

$\doteq 4.980$

$\doteq 4.98$ hours

$\therefore$ the half-life of caffeine is 4.98 hours http://www.timhortons.com/ca/en/menu/nutrition-and-wellness-resources.php

*Internet "Facts"*

| Tim Horton's Coffee size (Original Blend) | | vs. | Caffeine content |
|---|---|---|---|
| small | (10 oz.) | | 140 mg |
| medium | (14 oz.) | | 205 mg |
| large | (20 oz.) | | 270 mg |
| X-large | (24 oz.) | | 330 mg |

Starbucks

venti (20 oz.) **410 mg**

For comparison, a 8.4 oz can of Red Bull has 80 mg.

For comparison, a 12 oz can of Coca-Cola has 34 mg.

2.83 mg/oz

9.5 mg/oz

13.5-14.6 ? mg/oz

20.5 mg/oz