

Last day's Work: Begin Review p. 510

#1c, 2d, 3d*graph too, 4, 5b, 6b, 8*, 9, 10abc, 11c, 12, 13

*the answer for 8d is 3. c ab

*the answer for 11c is 2.552

p. 510 8. Use the laws of logarithms to evaluate.

a) $\log_6 42 - \log_6 7$

b) $\log_3 5 + \log_3 18 - \log_3 10$

c) $\log_7 \sqrt[3]{49}$

d) $2 \log_4 8$

$$\begin{aligned} \text{c) } \log_7 \sqrt[3]{49} &\rightarrow \log_7 (49^{\frac{1}{3}}) \\ &= \log_7 (7^2)^{\frac{1}{3}} \\ &= \frac{2}{3} \end{aligned}$$

p. 510

12. Solve.

a) $4^x + 6(4^{-x}) = 5$

b) $8(5^{2x}) + 8(5^x) = 6$

12a) $4^x + 6(4^{-x}) = 5$

$4^x(4^x) + 4^{-x}(6(4^x)) = 4^x(5)$

$4^{2x} + 6(4^0) = 5(4^x)$

$4^{2x} + 5(4^x) + 6 = 0$

Let $w = 4^x \therefore w^2 - 5w + 6 = 0$

$(w-3)(w-2) = 0$

$\therefore (4^x - 3)(4^x - 2) = 0$

$4^x = 3 \quad 4^x = 2$

$x = \frac{\log 3}{\log 4} \quad \text{or} \quad x = \frac{\log 2}{\log 4}$

$\approx 0.7924 \quad = \frac{1}{2}$

≈ 0.792

$$\begin{aligned} \text{12b) } 8(5^{2x}) + 8(5^x) &= 6 \\ \text{let } w = 5^x \therefore 8w^2 + 8w - 6 &= 0 \\ \therefore 4w^2 + 4w - 3 &= 0 \\ (2w + 3)(2w - 1) &= 0 \\ \therefore 2(5^x) + 3 = 0 \quad \text{or} \quad 2(5^x) - 1 &= 0 \\ 5^x = -\frac{3}{2} \quad 5^x = \frac{1}{2} \\ \text{Not possible} \quad \text{or} \quad x = \frac{\log \frac{1}{2}}{\log 5} \\ \text{to get -ve} \\ \therefore &= -0.4306 \\ &= -0.431 \end{aligned}$$

p. 510 13. The half-life of a certain substance is 3.6 days. How long will it take for 20 g of the substance to decay to 7 g?

13) half-life = 3.6 days

time for 20g $\rightarrow 7g$

$M(t) = 20\left(\frac{1}{2}\right)^{\frac{t}{3.6}}$

$7 = 20\left(\frac{1}{2}\right)^{\frac{t}{3.6}}$

$\frac{7}{20} = \frac{1}{2}^{\frac{t}{3.6}}$

$\log \frac{7}{20} = \frac{t}{3.6} \log \frac{1}{2}$

$\therefore 3.6 \log \left(\frac{7}{20}\right) = t$

$\log \frac{1}{2}$

$t = 5.452$

$\approx 5.45 \text{ days}$

8.6 Solving Logarithmic Equations



"I can solve and check any equation with logarithmic expressions. I also know what the restrictions are in an equation."

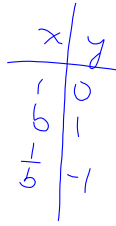
Recall: Given: $y = \log_b x$.

(Refer to p.450)

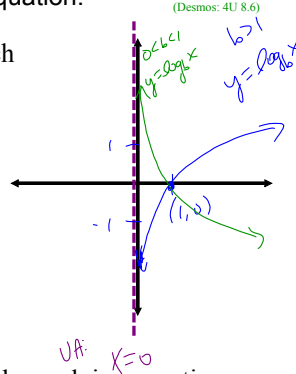
$D: \{x \in \mathbb{R} \mid x > 0\}$

$R: \{y \in \mathbb{R}\}$

$b > 0, b \neq 1$



Sketch



Since the domain of $y = \log_b x$ is $D: \{x \in \mathbb{R} \mid x > 0\}$, when solving equations involving logarithms, one must always identify the restrictions on x .

It is best to do this **as soon as possible**.

For example, if one is asked to solve $\log_5(2x - 5) = \log_5 7$, then whatever the final solution is for x , it must be greater than $\frac{5}{2}$.

Ex. 1: Solve: $\log_5(2x - 5) = \log_5 7$

$\therefore 2x - 5 = 7$
 $2x = 7 + 5$
 $2x = 12$
 $x = 6$

Rest. $2x - 5 > 0$
 $2x > 5$
 $x > \frac{5}{2}$

Identities:
 $b^m = b^n$
 $\therefore m = n$
 $\log_b m = \log_b n$
 $\therefore m = n$

(Check vs Restriction)
 $\therefore 6 > \frac{5}{2} \checkmark$
 $\therefore x = 6$ is the solution.

Ex. 2: Solve:

a) $\log_3(x - 4) + \log_3 5 = \log_3 10$

Restriction: $x - 4 > 0$
 $x > 4$

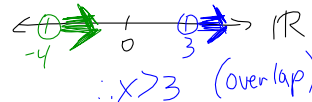
$\log_3[(x - 4)5] = \log_3 10$

$5(x - 4) = 10$
 $5x - 20 = 10$
 $5x = 10 + 20$
 $5x = 30$
 $x = 6$

(Check vs Restriction) $2^3 = 4$
 $\therefore 6 > 4 \checkmark$
 $\therefore x = 6$ is the solution.

b) $\log_2(x - 3) + \log_2(x + 4) = 3$

R: $x - 3 > 0$ AND $x + 4 > 0$
 $x > 3$ AND $x > -4$



$\log_2(x - 3)(x + 4) = 3$

$2^3 = (x - 3)(x + 4)$

$8 = x^2 + x - 12$

$0 = x^2 + x - 12 - 8$

$= x^2 + x - 20$

$= (x + 5)(x - 4)$

$x = -5$ or $x = 4$

$\therefore -5 \not> 3$ $\therefore 4 > 3 \checkmark$

$\therefore x = -5$ is an extraneous root (inadmissible)
 $\therefore x = 4$ is the solution.