8.7 Solving Problems with Exponential and Logarithmic Functions

"I can solve problems based on *applications*
of exponential and logarithmic functions.
I can apply what I have learned in unfamiliar settings."

Last day's Work: pp. 491–492 #1de, 2de, 4de, 5cef, 7bdef, 9, 12, 13, 14, 20
ab

Also **READ** pp. 494–499 (For Today)

Today's Entertainment: pp.499–501 #1 to 4, 5ab, 6ab, 7, 10, 13, 14*, 15
*#14) 7.5 years

p. 491 **7.** Solve.

a) $\log_7(x + 1) + \log_7(x - 5) = 1$
b) $\log_3(x - 2) + \log_3 x = 1$
c) $\log_6 x - \log_6(x - 1) = 1$
d) $\log(2x + 1) + \log(x - 1) = \log 9$
e) $\log(x + 2) + \log(x - 1) = 1$
f) $3 \log_2 x - \log_2 x = 8$

7f) $3\log_2 x - \log_2 x = 8 \quad \therefore x > 0$

$\therefore \log_2 x^3 - \log_2 x = \log_2 256$

$\log_2\left(\frac{x^3}{x}\right) = \log_2 256$

$\therefore \frac{x^3}{x} = 256$

$\log_2\left(\frac{x^3}{x}\right) = 8$

$\log_2 x^2 = 8$

$2^8 = x^2$

$256 = x^2$

$x^2 = 256$

$x = \pm 16$

$x = -16 \quad \therefore x = 16$

extraneous

p. 492 **9.** The loudness, L, of a sound in decibels (dB) can be calculated using the formula $L = 10 \log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound in watts per square metre (W/m^2) and $I_0 = 10^{-12}$ W/m^2.

a) A teacher is speaking to a class. Determine the intensity of the teacher's voice if the sound level is 50 dB.
b) Determine the intensity of the music in the earpiece of an MP3 player if the sound level is 84 dB.

9. $L = 10\log\left(\frac{I}{I_0}\right)$

a) $50 = 10\log\left(\frac{I}{10^{-12}}\right)$

$5 = \log I - \log 10^{-12}$

$\log 100000 + \log 10^{-12} = \log I$

$\log 100\,000 - 12\log 10 = \log I$

$5 - 12(1) = \log I$

$-7 = \log I$

$10^{-7} = I$

b) $84 = 10(\log I - \log 10^{-12})$

$8.4 = \log I + 12\log 10$

$8.4 - 12(1) = \log I$

$-3.6 = \log I$

$10^{-3.6} = I$

p. 492 **12.** Solve $\log_5(x-1) + \log_5(x-2) - \log_5(x+6) = 0$.

12) $\log_5(x-1) + \log_5(x-2) - \log_5(x+6) = 0$

$x>1 \wedge x>2 \wedge x>-6 \therefore x>2$

$\log_5 \boxed{\frac{(x-1)(x-2)}{(x+6)}} = 0$

$\therefore 5^0 = \frac{(x-1)(x-2)}{x+6}$

$(1)(x+6) = x^2 - 3x + 2$

$0 = x^2 - 3x - x + 2 - 6$

$= (x^2 - 4x - 4)$

$\to 0 = x^2 - 4x + 4 - 4 - 4$

$= (x-2)^2 - 8$

$8 = (x-2)^2$

$\therefore x = 2 \pm \sqrt{8}$ or $x = 2 \pm 2\sqrt{2}$

$x = 2 - \sqrt{8}$ extraneous

$\therefore x = 2 + \sqrt{8} \doteq 4.828$

$\doteq 4.83$

$\log_5 A = 0$

$5^0 = A$

$1 = A$

p. 492 **13.** Explain why there are no solutions to the equations $\log_3(-8) = x$ and $\log_{-3}9 = x$.

13. Why No Solution?

$\log_3(-8) = x$

$\therefore 3^x = -8$

No x will make -ve

$\log_{-3} 9 = x$

$\therefore -3^x = 9$

No x will make the

Not Same as $(-3)^x = 9$

-5^2

$= -(5)(5)$

$= -25$

$(-5)^2$

$= (-5)(-5)$

$= 25$

p. 492 **14.** a) Without solving the equation, state the restrictions on the variable x in the following: $\log(2x - 5) - \log(x - 3) = 5$

b) Why do these restrictions exist?

14a) $\log(2x-5) - \log(x-3) = 5$

$x > \frac{5}{2} \cap x > 3$

$\therefore x > 3$

p. 492 **20.** If $\left(\frac{1}{2}\right)^{x+y} = 16$ and $\log_{x-y}8 = -3$, calculate the values of x and y.

20) $\left(\frac{1}{2}\right)^{x+y} = 16$, $\log_{x-y}8 = -3$

$\left(2^{-1}\right)^{x+y} = 2^4$, $(x-y)^{-3} = 8$

$\therefore -x-y = 4$ $\quad (x-y)^{-3} = \left(\frac{1}{2}\right)^{-3}$

$\therefore x-y = \frac{1}{2}$

$-x-y = 4$

$x-y = \frac{1}{2}$

$-2y = \frac{9}{2}$

$y = \frac{-9}{4}$

Sub in (2)

$x - \left(\frac{-9}{4}\right) = \frac{1}{2}$

$x = \frac{2}{4} - \frac{9}{4}$

$= \frac{-7}{4}$