

Last day's Work:

pp. 528-530 #1ce, 2, 3*, 7ac, 8, 10, 12, 16

*answer wrong in back...there is no domain, so it is impossible for $f-g$ to exist.

p. 528

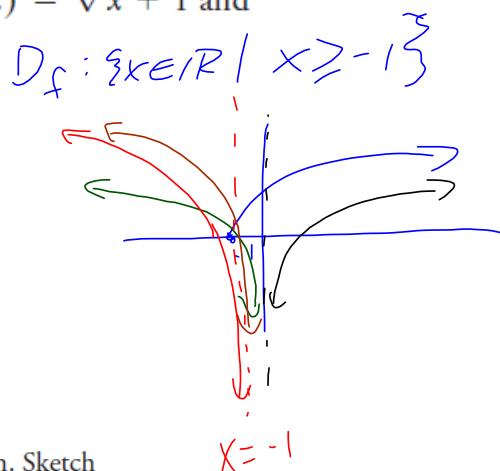
3. What is the domain of
- $f - g$
- , where
- $f(x) = \sqrt{x+1}$
- and

$$g(x) = \underbrace{2 \log[-(x+1)]}_{\text{red underline}}$$

$$D_g : \{x \in \mathbb{R} \mid x < -1\}$$

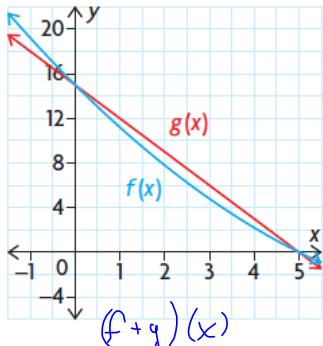
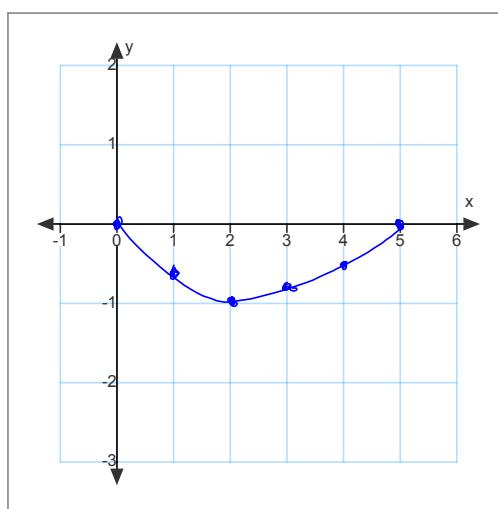
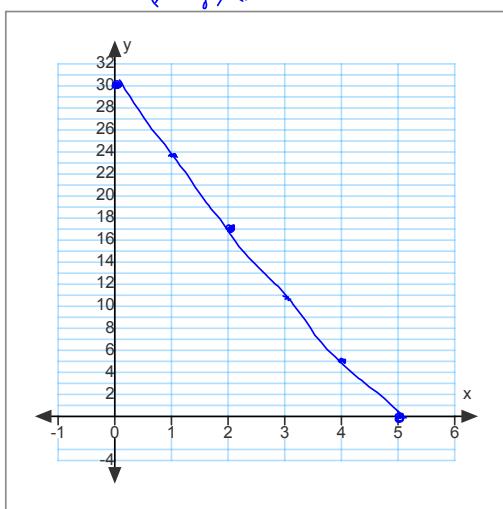
\therefore No Overlap

$\therefore f-g$ is NOT Possible



p. 529

8. The graphs of
- $f(x)$
- and
- $g(x)$
- , where
- $0 \leq x \leq 5$
- , are shown. Sketch the graphs of
- $(f+g)(x)$
- and
- $(f-g)(x)$
- .

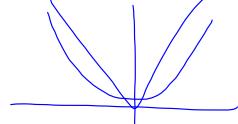

 $(f+g)(x)$


9.3_4 Combining Two Functions - Products and Quotients (F2018e-p)

January 16, 2019

- p. 530 10. a) Is the sum of two even functions even, odd, or neither? Explain.
 b) Is the sum of two odd functions even, odd, or neither? Explain.
 c) Is the sum of an even function and an odd function even, odd, or neither? Explain.

a) $y = x^2$ vs. $y = |x|$



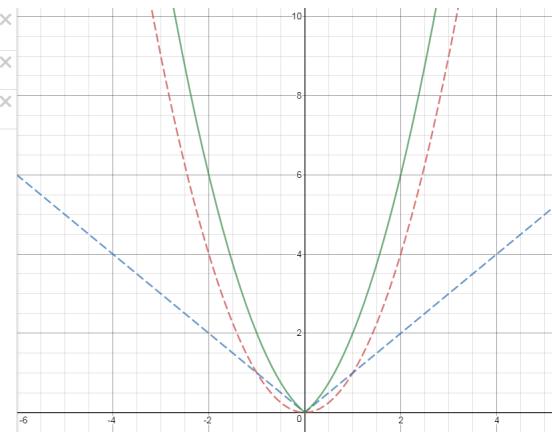
EVEN
 $f(-x) = f(x)$

Odd
 $f(-x) = -f(x)$

- 1 $f(x) = x^2$
 2 $g(x) = \text{abs}(x)$
 3 $a(x) = f(x) + g(x)$

Result:

$$\begin{aligned} &\text{EVEN} + \text{EVEN} \\ &= \text{EVEN} \end{aligned}$$

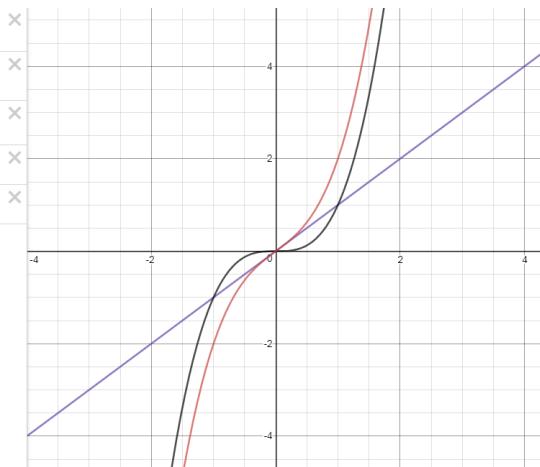


b) $y = x$ vs. $y = x^3$

- 1 $o_1(x) = x$
 2 $o_2(x) = x^3$
 3 $o_3(x) = o_1(x) + o_2(x)$

Result:

$$\text{Odd} + \text{Odd} = \text{Odd}$$



b) $y = x^4$ vs. $y = x$

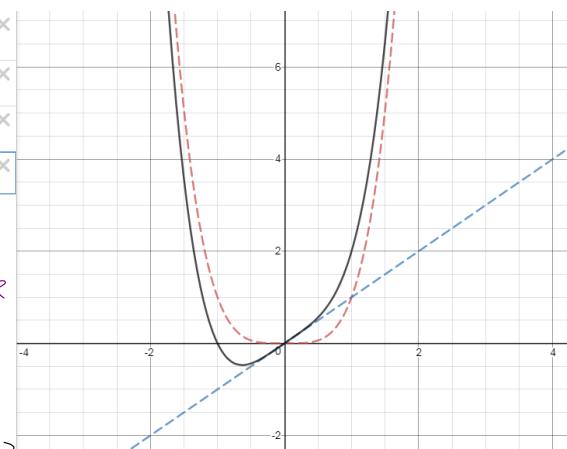
- 1 $f(x) = x^4$
 2 $g(x) = x$
 3 $a(x) = f(x) + g(x)$

Result:

$$\text{EVEN} + \text{ODD} = \text{NEITHER}$$

(There is no rotational symmetry for $a(x) \rightarrow$

Symmetry for $a(x) \rightarrow$
 black graph \Rightarrow



- p. 530 16. Let $f(x) = x^2 - nx + 5$ and $g(x) = mx^2 + x - 3$. The functions are combined to form the new function $h(x) = f(x) + g(x)$. Points $(1, 3)$ and $(-2, 18)$ satisfy the new function. Determine the values of m and n .

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= x^2 - nx + 5 + mx^2 + x - 3 \\ &= x^2 + mx^2 + x - nx + 5 - 3 \\ &= x^2(1+m) + x(1-n) + 2 \end{aligned}$$

$$h(1) = (1)^2(1+m) + (1)(1-n) + 2$$

$$3 = 1+m+1-n+2$$

$$3 = m-n+4$$

$$-1 = m-n \quad \textcircled{1}$$

$$h(1) = 3$$

$$h(-2) = 18$$

$$h(-2) = (-2)^2(1+m) + (-2)(1-n) + 2$$

$$18 = 4(1+m) - 2 + 2n + 2$$

$$= 4 + 4m + 2n$$

$$14 = 4m + 2n$$

$$7 = 2m + n \quad \textcircled{2}$$

$$-1 = m-n \quad \textcircled{1}$$

$$7 = 2m + n \quad \textcircled{2}$$

$$\begin{array}{r} -1 = m-n \\ 7 = 2m+n \\ \hline 6 = 3m \end{array} \quad \textcircled{1} + \textcircled{2}$$

$$m = 2$$

Sub in \textcircled{1} $\therefore -1 = (2) - n$

$$\therefore n = 3$$

Extra: $\therefore f(x) = x^2 - 3x + 5$

$g(x) = 2x^2 + x - 3$

9.3 and 9.4 Combining Two Functions: Products and Quotients



"I can multiply and divide functions. I know their main properties.
I can apply what I have learned in unfamiliar settings."

Given functions $f(x)$ and $g(x)$:

The **product of two functions** is $f(x) \cdot g(x)$ which can also be written as $(f \times g)(x)$.

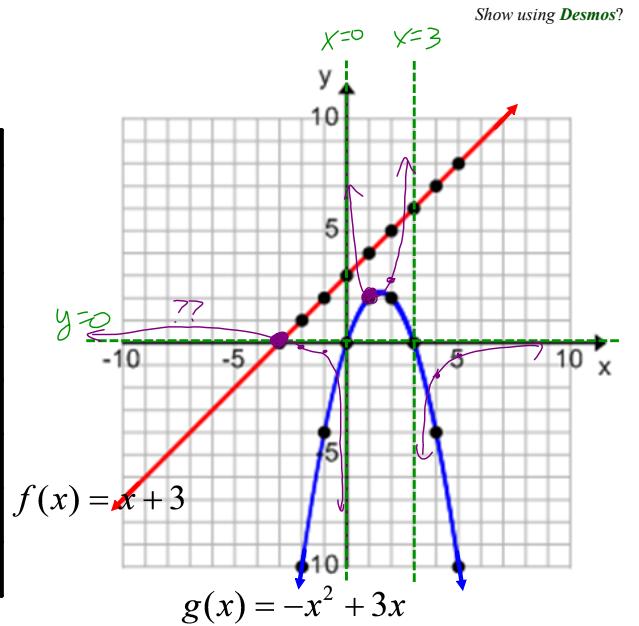
The **quotient of two functions** is $f(x) \div g(x)$

which can also be written as $(f \div g)(x)$, where $g(x) \neq 0$.

Ex. 1:

Complete the table of values, then graph $(f \div g)(x)$

x	$f(x)$	$g(x)$	$(f \times g)(x)$	$(f \div g)(x)$
-2	1	-10	-10	$\frac{1}{-10} = -0.1$
-1	2	-4	-8	$\frac{2}{-4} = -0.5$
0	3	0	0	$\frac{3}{0} = \text{undefined}$
1	4	2	8	$\frac{4}{2} = 2$
2	5	2	10	$\frac{5}{2} = 2.5$
3	6	0	0	$\frac{6}{0} = \text{undefined}$
4	7	-4	-28	$\frac{7}{-4} = -1.75$
5	8	-10	-80	$\frac{8}{-10} = -0.8$



The **domain** of:

$(f \times g)(x)$ is the intersection of the domains of $f(x)$ and $g(x)$.

$$D_{f \times g} : \{x \in \mathbb{R}\}$$

$(f \div g)(x)$ is the intersection of the domains of $f(x)$ and $g(x)$

except in the case where $g(x) = 0$.

$$D_f : \{x \in \mathbb{R}\} \quad D_g : \{x \in \mathbb{R}\}$$

$$D_{f \div g} : \{x \in \mathbb{R} \mid x \neq 0, 3\}$$

Ex. 2: State the domain of $(f \div g)(x)$ where $f(x) = 3x^2$ and $g(x) = x^3 - 4x$.

$$\begin{aligned} f(x) &= \frac{f(x)}{g(x)} \\ &= \frac{3x^2}{x^3 - 4x} \\ &= \frac{3x^2}{x(x-2)(x+2)} \\ &= \frac{3x}{(x-2)(x+2)} \end{aligned}$$

$\therefore x=0, 2, -2$
points of discontinuity

$$= \frac{3x}{(x-2)(x+2)}$$

$D_{f \div g} : \{x \in \mathbb{R} \mid x \neq 0, 2, -2\}$

: hole at $x=0$

$x=0$ But JA $x = -2, x = 2$

Practice: pp. 537-538 #4bd, 5bd, 8ab, 10 AND p.542 #1bc, 2 (just for 1bc).

$$f(x) = x + 3$$

$$g(x) = -x^2 + 3x$$

$$\begin{aligned} h(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x+3}{-x^2 + 3x} \\ &= \frac{x+3}{-x(x-3)} \end{aligned}$$

Restrictions: $x \neq 0, 3$

V.A.: $x = 0, x = 3$

Holes: None

H.A.: $\frac{0}{-1}$

$$\therefore y = 0$$

Zeros: $0 = \frac{x+3}{-x(x-3)}$

$$\therefore 0 = x + 3$$

$$\therefore x = -3$$

$y\text{-int: let } x = 0 \text{ (rest:)}$
But $x \neq 0$
 $\therefore \text{No } y\text{-int}$

$$f(x) = x + 3 \quad g(x) = -x^2 + 3x$$

$$P(x) = (f \cdot g)(x)$$

$$= (x+3)(-x^2 + 3x)$$

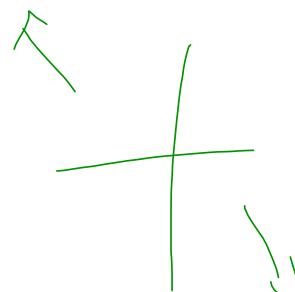
$$= -x^3 + 3x^2 - 3x^2 + 9x$$

$$= -x^3 + 9x$$

$$= -x(x^2 - 9)$$

$$= -x(x-3)(x+3)$$

<u>zeros</u>	<u>order</u>
0	1
3	1
-3	1



Recall:

(from Unit 5)

$$y = \frac{ax+b}{cx+d}$$

$$\text{HA} : y = \frac{a}{c}$$

$$y = \frac{ax+b}{cx^2+dx+e} = \frac{0x^2+ax+b}{cx^2+dx+e}$$

$$\text{HA} : y = \frac{0}{c} = 0$$