Last day's Work:

pp. 537-538 #4bd, 5bd, 8ab, 10 **AND** p.542 #1bc, 2 (just for 1bc).

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4. Determine $(f \times g)(x)$ for each of the following pairs of functions.

K a) $f(x) = x - 7, g(x) = x + 7$

b) $f(x) = \sqrt{x + 10}, g(x) = \sqrt{x + 10}$

d) $f(x) = -4x - 7, g(x) = 4x + 7$

4b) $f(x) = \sqrt{x+10}, g(x) = \sqrt{x+10}$

$(f \times g)(x)$

$= (\sqrt{x+10})(\sqrt{x+10})$

$= x+10$

d) $f(x) = -4x-7, g(x) = 4x+7$

$(f \times g)(x) = (-4x-7)(4x+7)$

$= -(4x+7)(4x+7)$

$= -16x^2 - 56x - 49$

$= -(4x+7)^2$

$= -(4(x+\frac{7}{4}))^2$

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5. For each of the problems in question 4, state the domain and range of $(f \times g)(x)$.

5b) $D_{f \times g}: \{x \in \mathbb{R} / x \geq -10\}$

$R_{f \times g}: \{y \in \mathbb{R} / y \geq 0\}$

5d) $D_{f \times g}: \{x \in \mathbb{R}\}$

$R_{f \times g}: \{y \in \mathbb{R} / y \leq 0\}$

$\sqrt{A} \times \sqrt{B}$

$= \sqrt{A \times B}$

$\sqrt{x} \cdot \sqrt{x}$

$= \sqrt{x \cdot x}$

$= \sqrt{x^2}$

$= x$

$\sqrt{9} \times \sqrt{4} \longrightarrow 3 \times 2$

$= 6$

$= \sqrt{9 \times 4}$

$= \sqrt{36}$

$= 6$

$\sqrt{9} \cdot \sqrt{9}$

$= \sqrt{9 \cdot 9}$

$= \sqrt{81}$

$= 9$

p. 538 **8.** For each of the following pairs of functions, state the domain of $(f \times g)(x)$.

a) $f(x) = \frac{1}{x^2 - 5x - 14}, g(x) = \sec x$

b) $f(x) = 99^x, g(x) = \log(x - 8)$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\frac{(2n-1)\pi}{2}, n \in I$

8a) $f(x) = \frac{1}{x^2-5x-14}$, $g(x) = \sec(x)$

$= \frac{1}{(x-7)(x+2)}$, $D_g: \{x \in R, x \neq \}$

$D_f: \{x \in R / x \neq 7, -2\}$

$\therefore D_{f \times g}: \{x \in R / x = 7, -2, \frac{(2n-1)\pi}{2}, n \in I\}$

8b) $f(x) = 99^x$, $g(x) = \log(x-8)$

$D_f: \{x \in R\}$, $D_g: \{x \in R / x > 8\}$

$\therefore D_{f \times g}: \{x \in R / x > 8\}$

p. 538 **10.** An average of 20 000 people visit the Lakeside Amusement Park each day in the summer. The admission fee is \$25.00. Consultants predict that, for each \$1.00 increase in the admission fee, the park will lose an average of 750 customers each day.

a) Determine the function that represents the projected daily revenue if the admission fee is increased.

b) Is the revenue function a product function? Explain.

c) Estimate the ticket price that will maximize revenue.

10a) Let x represent the number of \$1-dollar increases.

Revenue(x) = (Price per ticket)(Number of Tickets Sold)

$= (25 + 1x)(20\,000 - 750x)$

b) Yes, the revenue function is a product function, because it is the product of the ticket price function (which is linear) and the ticket sales function (which is also linear).

c) The Revenue function is quadratic, so we need the x-coordinate of the vertex.

Let $R(x) = 0$

$\therefore 25 + x = 0$ or $20000 - 750x = 0$

$x = -25$ $\quad x = \frac{20\,000}{750}$

$= \frac{80}{3}$

$\therefore$ A of S: $x = \frac{-25 + \frac{80}{3}}{2}$

$= \frac{5}{6}$ (of a dollar)

$\doteq 0.833$

$\doteq \$0.83$ or $83¢$

$P(x) = 25 + x$

$\doteq \$25.83$ is the price which will maximize revenue.

9.5 Combining Two Functions: Composition

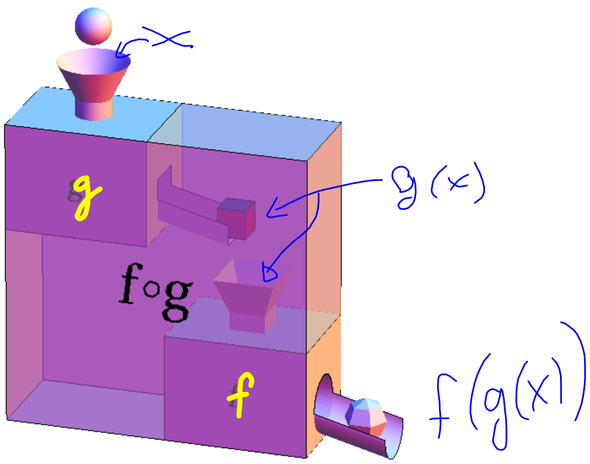
"I can compose functions. I know a composition's main properties.
I can apply what I have learned in unfamiliar settings."

Recall: a function takes any one input over its domain, and produces only one output in its range.

$$x \longrightarrow \boxed{f} \longrightarrow f(x)$$

To <u>**compose**</u> a function means to apply the output of one function (**inner function**) as the input of another function (**outer function**).

Let the inner function be $g(x)$ and let the outer function be $f(x)$.



Note: **the result is still a function!**

This is stated as "the composition of f with g". It can also be stated as: "f composed with g".

$$f(g(x)) = (f \circ g)(x)$$

Ex. 1:
State two algebraic representations of "the composition of g with f".

$$g(f(x)) = (g \circ f)(x)$$

Ex. 2:

Given: $f(x)=\sqrt{x-4}$ and $g(x)=2x+8$

Find, in its simplest form:

a) $f\circ g = f(g(x))$

$= f(2x+8)$

$= \sqrt{(2x+8)-4}$

$= \sqrt{2x+4}$ ← Simplified

$= \sqrt{2(x+2)}$ ← Simplified Factored

b) $g\circ f = g(f(x))$

$= g(\sqrt{x-4})$

$= 2(\sqrt{x-4})+8$

$= 2\sqrt{x-4}+8$

$= 2(\sqrt{x-4}+4)$

Ex. 3:

Find $g(f(-2))$, where $f(x) = 3x + 8$ and $g(x) = 2x^2 - 12$.

If time, "verify the long way" on next screen.

$f(-2)=3(-2)+8$

$=-6+8$

$=2$

$g(f(-2))$

$=g(2)$

$=2(2)^2-12$

$=2(4)-12$

$=8-12$

$=-4$

Ex. 4:

If $f(x) = \log(x-5)$ and $g(x) = 3x - 4$, find the domain of $f\circ g$ algebraically.

Graph $(f\circ g)(x)$.

Show using ***Desmos****?*

$=f(g(x))$

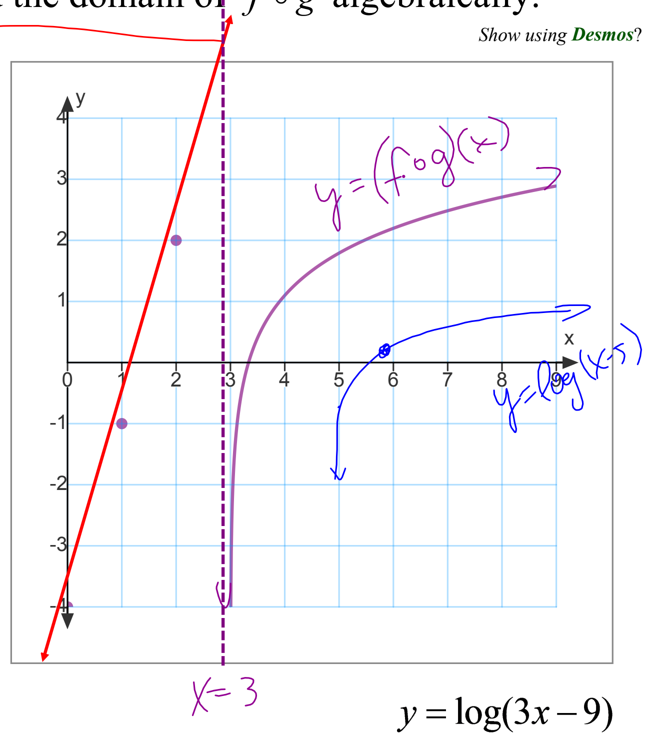
$=f(3x-4)$

$=\log((3x-4)-5)$

$=\log(3x-9)$

$=\log(3(x-3))$

$D_{f\circ g}: \{x\in\mathbb{R} \mid x>3\}$



$y=\log(3x-9)$

READ: p. 551

Entertainment pp. 552-553 #1bd, 2ad, 3bc, 5aef, 6ac, 7df

Ex. 3: (*The long way*: compose the functions algebraically, then substitute.)

Find $g(f(-2))$, where $f(x) = 3x + 8$ and $g(x) = 2x^2 - 12$.

$g(f(x)) = 2(3x + 8)^2 - 12$

$= 2(9x^2 + 48x + 64) - 12$

$= 18x^2 + 96x + 128 - 12$

$= 18x^2 + 96x + 116$

$g(f(-2)) = 18(-2)^2 + 96(-2) + 116$

$= 18(4) - 192 + 116$

$= 72 - 76$

$= -4$

$g(f(-2)) = 2(3x + 8)^2 - 12$

$= 2(3(-2) + 8)^2 - 12$

$= 2(-6 + 8)^2 - 12$

$= 2(-2)^2 - 12$

$= 2(4) - 12$

$= 8 - 12$

$= -4$