

Last day's Work: pp. 552-553 #1bd, 2ad, 3bc, 5aef, 6ac, 7df

Optional Ch.9 Review: pp. 576-577 #2-10

- p. 552 5. In each case, functions f and g are defined for $x \in \mathbb{R}$. For each pair of functions, determine the expression and the domain of $f(g(x))$ and $g(f(x))$. Graph each result.
- a) $f(x) = 3x^2$, $g(x) = x - 1$

$$\begin{aligned}
 & 5a) f(x) = 3x^2, g(x) = x - 1 \\
 & f(g(x)) = 3(x-1)^2 \quad (g \circ f)(x) \rightarrow \begin{array}{c} \text{Graph of } f(x) = 3x^2 \\ \text{Graph of } g(x) = x-1 \end{array} \\
 & D_{f \circ g} = \{x \in \mathbb{R}\} \\
 & g(f(x)) = (3x^2) - 1 \quad \leftarrow \begin{array}{c} \text{Graph of } g(x) = x-1 \\ \text{Graph of } f(x) = 3x^2 \end{array} \\
 & = 3x^2 - 1 \\
 & D_{g \circ f} = \{x \in \mathbb{R}\}
 \end{aligned}$$

p. 552 6. For each of the following,

- determine the defining equation for $f \circ g$ and $g \circ f$
- determine the domain and range of $f \circ g$ and $g \circ f$

- a) $f(x) = 3x, g(x) = \sqrt{x-4}$ d) $f(x) = 2^x, g(x) = \sqrt{x-1}$
 b) $f(x) = \sqrt{x}, g(x) = 3x+1$ e) $f(x) = 10^x, g(x) = \log x$
 c) $f(x) = \sqrt{4-x^2}, g(x) = x^2$ f) $f(x) = \sin x, g(x) = 5^{2x} + 1$

6a) $f(x) = 3x, g(x) = \sqrt{x-4} \quad D_g : \{x \in \mathbb{R} \mid x \geq 4\}$

$$(f \circ g)(x) = \sqrt{3x-4}$$

$$D_{f \circ g} : \{x \in \mathbb{R} \mid x \geq 4\} \quad D_{g \circ f} : \{x \in \mathbb{R} \mid x \geq \frac{4}{3}\}$$

$$R_{f \circ g} : \{y \in \mathbb{R} \mid y \geq 0\} \quad R_{g \circ f} : \{y \in \mathbb{R} \mid y \geq 0\}$$

6c) $f(x) = \sqrt{4-x^2}, g(x) = x^2$

$$D_f : \{x \in \mathbb{R} \mid -2 \leq x \leq 2\} \quad D_g : \{x \in \mathbb{R}\} \quad R_g : \{y \in \mathbb{R} \mid y \geq 0\}$$

$$(f \circ g)(x) = \sqrt{4-(x^2)^2}$$

$$= \sqrt{4-x^4}$$

$$\therefore 4-x^4 \geq 0 \quad \left. \begin{array}{l} \therefore D_{f \circ g} : [-\sqrt[4]{2}, \sqrt[4]{2}] \\ x^4 \leq 4 \end{array} \right\}$$

$$\left. \begin{array}{l} (x^2)^2 \leq 2^2 \\ \therefore x^2 \leq 2 \end{array} \right\} \therefore R_{f \circ g} : [0, 2]$$

$$\therefore |x| \leq \sqrt{2}$$

Refer to p. 551 "Need to know"

... domain of $f \circ g$ is subset

of D_g

6c) $(g \circ f)(x) = (\sqrt{4-x^2})^2$

$$= 4-x^2$$

$$= -x^2 + 4 \quad \leftarrow D : (-\infty, \infty) \quad R : [-\infty, 4]$$

$$\therefore D_f : [-2, 2]$$

$$\therefore D_{g \circ f} : [-2, 2]$$

$$\therefore R_{g \circ f} : [0, 4]$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 2^2$$

$$x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

$$y = \pm \sqrt{4 - x^2}$$