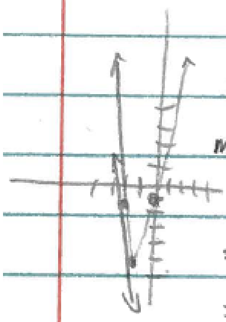
Wkst #2

3. Sketch the graph of each of the following functions. Draw a secant line on each sketch that you could use to estimate the slope of the tangent when $x = -2$. Use the results to estimate the slope of the line tangent to each of the functions when $x = -2$.
   a) $f(x) = 4x^2 + 8x - 1$

3a) $f(x) = 4x^2 + 8x - 1$

$= 4(x^2 + 2x + 1 - 1) - 1$

$= 4(x+1)^2 - 5$



use $f(-2) = -1$

$f(-2.01) = -0.9196$

$m = \dfrac{-0.9196 - (-1)}{-2.01 - (-2)}$

$= \dfrac{0.0804}{-0.01}$

$= -8.04$

$\boxed{\text{* Exact } m = -8}$

4. Matthew leaves his house to go for a run. He ran at a constant speed of 10 km/h for 30 minutes. The path sloped upward, so he ran at a rate of 6 km/h for 15 minutes. At the top of the hill, he increased his speed to 8 km/h for 30 minutes. He stopped for 15 minutes to rest. Finally, he ran at a constant speed of 10 km/h for 30 minutes.

   a) Sketch a graph of Matthew's distance versus time while running.
   b) What is Matthew's average speed during the entire time that he ran?
   c) What is Matthew's average speed between 15 and 45 minutes?
   d) What is Matthew's instantaneous speed at 60 minutes?

4a) 10km ; 30 min = 5 km

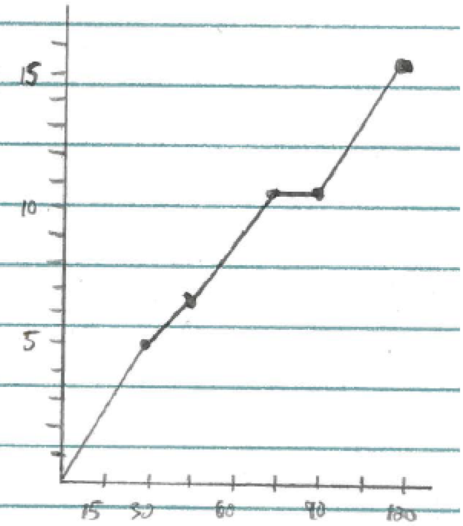
6 ; 15 = 1.5 km

8 ; 30 = 4 km

0 ; 15 = 0

10 ; 30 = 5 km

4a)



4b) avg. speed

$= \dfrac{15.5 \text{ km}}{2 \text{ h}}$

$= 7.75$ km/h

c) [15, 45]

avg. speed $= \dfrac{6.5 - 2.5}{45 - 15}$

$= \dfrac{4 \text{ km}}{30 \text{ min}}$

$= \dfrac{4 \text{ km}}{0.5 \text{ hr}}$

$= 8$ km/h

d) between 45 min & 75 min, speed = 8 km/h

∴ instantaneous at 60 min is 8 km/h

Wkst #5

5. Solve each equation algebraically. Check your answers.

a) $0 = \frac{x-3}{500}$

b) $\frac{x+3}{7} = 10$

c) $\frac{5x+2}{3} = 6 - x$

d) $-\frac{4}{2x+3} = 1(-2+3x)$

e) $\frac{4}{5x} = \frac{6}{2x-1}$

d) Rest: $x = -\frac{3}{2}$

$$\frac{-4}{2x+3} = -2+3x$$

$$-4 = (-2+3x)(2x+3)$$

$$0 = -4x - 6 + 6x^2 + 9x + 4$$

$$0 = 6x^2 + 5x - 2$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(6)(-2)}}{2(6)}$$

$$= \frac{-5 \pm \sqrt{25+48}}{12}$$

$x = \frac{-5+\sqrt{73}}{12}$ or $x = \frac{-5-\sqrt{73}}{12}$

$\doteq 0.2953$ $\quad\doteq -1.1286$

$\doteq 0.295$ $\quad\doteq -1.129$

8. Estimate the slope of the line tangent to the function at the given value of x. At what point(s) is it not possible to do so?

a) $f(x) = \frac{x+10}{x-20}$, $x = 30$

8c* (* you MUST use "first principles" and algebraically determine the EXACT value of the instantaneous rate of change),

c) $f(x) = \frac{2x}{3x+4}$, $x = -2$

$$\text{iroc} = \frac{f(x+h) - f(x)}{h}, \ h \to 0$$

$$= \frac{\frac{2(-2+h)}{3(-2+h)+4} - \frac{2(-2)}{3(-2)+4}}{h}, \text{ as } h \to 0$$

$$= \left[\frac{-4+2h}{-6+3h+4} - \frac{-4}{-6+4}\right] \div h, \text{ as } h \to 0$$

$$= \left[\frac{2h-4}{3h-2} + \frac{4}{-2}\right] \times \frac{1}{h}, \text{ as } h \to 0$$

$$= \left[\frac{2h-4}{3h-2} - 2\left(\frac{3h-2}{3h-2}\right)\right] \times \frac{1}{h}, \text{ as } h \to 0$$

$$= \left[\frac{2h-4-6h+4}{3h-2}\right] \times \frac{1}{h}, \text{ as } h \to 0$$

$$= \frac{-4h}{3h-2} \times \frac{1}{h}, \text{ as } h \to 0$$

$$= \frac{-4}{3h-2}, \text{ as } h \to 0$$

$$= \frac{-4}{-2}$$

$$= 2$$

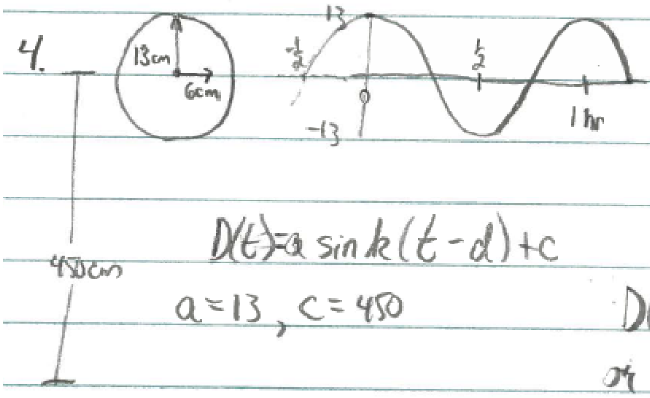
$\therefore$ the slope of the tangent at $x = -2$ is 2.

[Not possible to estimate at $x = -\frac{4}{3}$]

Wkst #6

4. A clock is hanging on a wall, with the centre of the clock 4.5 metres above the ground. Both the minute hand and the second hand are 13 cm long, while the hour hand is 6 cm long. Determine the equations of the sine function that describe the distance of the tip of each hand above the ground as a function of time. Assume that the time t is in hours and that the distance $D(t)$ is in cm. Also assume that at $t = 0$ it is 3 AM.

4.



$D(t) = a \sin k(t-d) + c$

$a = 13$, $c = 450$

Minute hand ; time in hours

$k = \frac{2\pi}{\text{period}}$ period = 1 h

$= \frac{2\pi}{1}$ $d = \frac{1}{4}$ h

$= 2\pi$

$D(t) = -13\sin\left(2\pi\left(t - \frac{1}{4}\right)\right) + 450$

or $D(t) = 13\sin\left(2\pi\left(t + \frac{1}{4}\right)\right) + 450$

Hour hand

$k = \frac{2\pi}{12}$ period = 12 h

$d = 0$ (std. pos.)

$= \frac{\pi}{6}$

$\therefore D(t) = -6\sin\left(\frac{\pi}{6}t\right) + 450$

Second hand ; time in hours

$k = \frac{2\pi}{\frac{1}{60}}$ period = $\frac{1}{60}$ of hour

$= 120\pi$ $d = \frac{1}{4}$ of $\frac{1}{60}$ of hour

$= \frac{1}{240}$ of hour

$\therefore D(t) = -13\sin\left(120\pi\left(t - \frac{1}{240}\right)\right) + 450$

or $= 13\sin\left(120\pi\left(t + \frac{1}{240}\right)\right) + 450$

Ch.3 Self-test #1a
p.186

1. a) Write the standard form of a general polynomial function. Then state the degree and leading coefficient of this function.
   b) What is the greatest number of turning points that this function can have?
   c) What is the greatest number of zeros that this function can have?
   d) If the least number of zeros is one, describe the degree of this function.
   e) A polynomial function is less than zero for all x. Describe the degree and the leading coefficient of this function.

Chapter Self-Test, p. 186

1. a) $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number. The degree of the function is n; the leading coefficient is a_n.
$y = x^4 - 2x^3 + x^2 - 2x + 8$.
The degree of this function is 4, and the leading coefficient is 1.
b) The maximum number of turning points is $n - 1$.
c) The function may have at most n zeros (the same as the degree).
d) If the least number of zeros is one, it is an odd degree function.
e) The function is an even degree function with a negative leading coefficient.

Ch.1 Self-test p.62

9. A certain tax policy states that the first \$50 000 of income is taxed at 5% and any income above \$50 000 is taxed at 12%.

a) Calculate the tax on \$125 000.

b) Write a function that models the tax policy.

$f(x) = 0.05x \qquad 0 \le x \le 50000$

$= 0.12(x - 50000) + 2500 \qquad x > 50000$

if 50 000

$T(50\,000) = 0.05(50000)$

$= 2500$

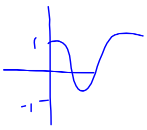
Wkst #9 5. For each of the following sets of functions, determine the domain and range of $f \circ g$ and $g \circ f$.

a) $f(x) = x^2 - 25$; $g(x) = \cos x$

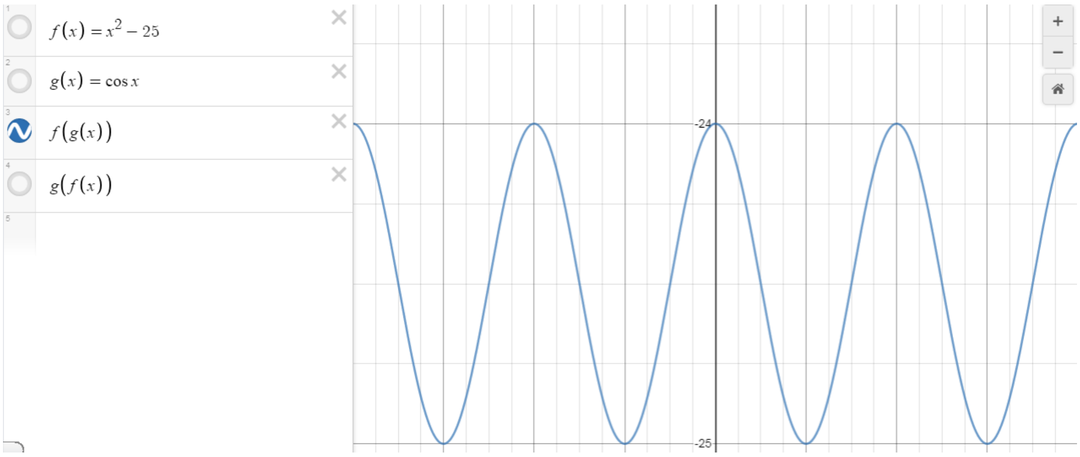
$(f \circ g)(x) = (\cos x)^2 - 25$

$= \cos^2 x - 25$

$D: \{x \in \mathbb{R}\}$



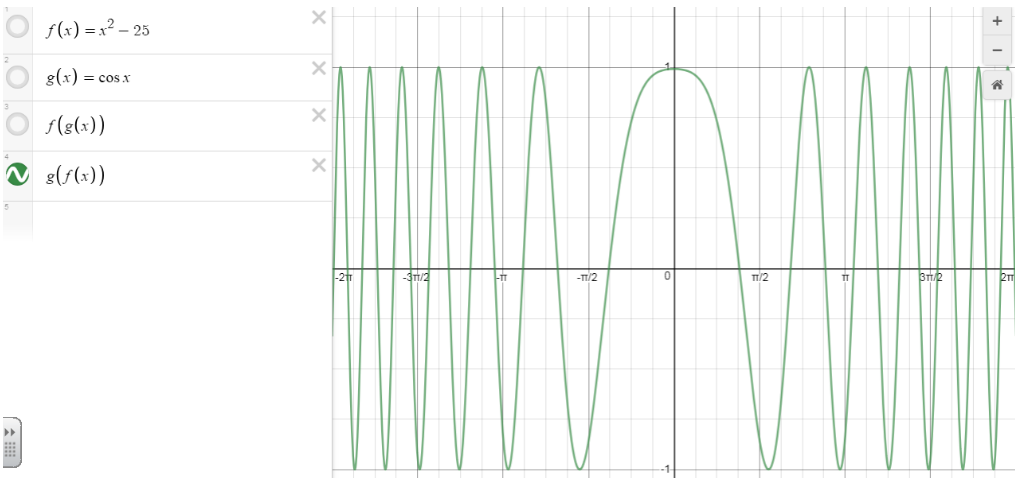
$R: \{y \in \mathbb{R} \mid -25 \leq y \leq -24\}$



$(g \circ f)(x) = \cos(x^2 - 25)$

$D: \{x \in \mathbb{R}\}$

$R: \{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$



Wkst #1

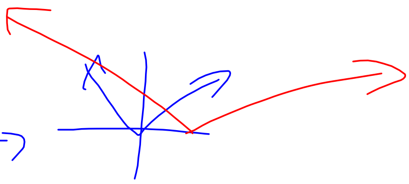
2. State the domain and range of each function.

a) $y = |x - 1|$

b) $y = -2|x + 3| - 4$

c) $y = \left|\frac{x - 3}{6}\right|$

↳ parent $f(x) = |x|$ →



$D_f: \{x \in \mathbb{R}\}$ $R_f: \{y \in \mathbb{R} \mid y \geq 0\}$

$y = \left|\frac{1}{6}(x-3)\right|$ D R